THE FAILURE OF CORPORATE FAILURE MODELS TO CLASSIFY AND PREDICT: - ASPECTS AND REFINEMENTS.

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A Sobering Thought

"We all make decisions whose consequences are affected by future events. If we could forecast the future with accuracy, then we would look pretty good to our family, friends, and colleagues. So we become easy pigeons for a plethora of persuasive prognosticators who are eager to sell us their wares. Like the infamous snake oil salesman of the past, they ply their trade with deftness, and we are eager to gulp their concoctions,"

Lawrence S. Davidson, 1989, p.2

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Abstract

Much has been written about the use of multiple discriminant analysis in corporate distress classification and forecasting. Classification and prediction models are notoriously difficult to establish in such a way that they will stand the ultimate test of time. Many articles severely criticise the use of the technique yet there are aspects which may improve our ability to develop satisfactory models. We are probably yet a long way off from being able to do so with any great degree of satisfaction, yet it behoves us to try to develop models that do justice to the assumptions and the theory. This thesis explores several important aspects of the model-building process and concludes that some of the more conventional criticisms of the models developed so far are less important than claimed. It suggests that more critical than the failure to meet the conditions of multivariate normality, the equality of the variance-covariance matrices, and the use of *a priori* probabilities are the need for: a satisfactory model specification that can be theoretically justified, the strict use of random sampling, the efficient use of sample data, the search for stable mean vectors which are significantly different from each other, and *ex ante* validation. If these requirements are met then the MDA technique is robust enough to cope with breaches of the assumptions.

Chapter One

PROBLEMS IN CLASSIFYING AND FORECASTING COMPANY FAILURE WITH MULTIPLE LINEAR DISCRIMINANT ANALYSIS.

INTRODUCTION:

Since Beaver first published his study of corporate distress prediction or classification in the late 1960s and since Altman first published his work using multivariate linear discriminant functions two years later, many other multivariate models have been developed for a range of industries in different economies. Multiple linear discriminant analysis [MDA] has been extensively used in the process by many researchers who seem to have ignored important aspects and limitations of the technique. Several authors [e.g. Joy & Tollefson,1975; Eisenbeis,1977; Jones,1987] have published comprehensive articles, which, amongst other issues, outline some of the failures which are likely to occur if MDA is inappropriately used.

For over 20 years now multivariate discriminant analysis has been used to build models from which we have attempted to classify and/or forecast company failure. While published models usually discriminate well between those companies which fail and those which remain in business, they are usually less successful when tested against a "*hold-out*" sample. When tested against other samples from the same population, or when evaluated in another time period they frequently fail dismally. Norusis [1986] discusses this issue, but offers no solutions to the problem. There are several reasons why seemingly satisfactory discriminant models of corporate failure themselves fail. In a review of the literature this chapter defines each of these issues in turn, and identifies several issues about which further research is needed. In subsequent chapters each issue is explored in detail.

The academic community is aware of the strengths and weaknesses of forecasting models of corporate distress as many of the issues have been discussed quite widely in the literature. As Jones, [1987], points out *"the preferred statistical technique has evolved from univariate to discriminant to logit analysis ......Researchers have noted that discriminant analysis, a popular multivariate technique, requires the assumptions of multivariate normality and equal covariances and that these assumptions are typically violated.,"* [p.156] A central focus of this thesis is that although there are frequent violations of the assumptions this may done under certain conditions and yet the technique may still work satisfactorily. We need to be able to understand and recognise the kinds of circumstances under which we may violate these assumptions and still produce valid discriminant models. We need to be sure that we do not over react in our critical evaluation of the use of discriminant models. If we examine the source of some of the failures, and if we clearly identify the issues involved then we can still use multiple linear discriminant analysis as a modelling technique to produce valid models of corporate distress. The cause of failure of MDA models is not so much in the discriminant methodology, although the valid application may be very exacting, but in its wrongful or naive usage or is sourced in the nature of the data itself, or more particularly in the manner in which models have been *specified*. If the source of the failure lies in the ratio data itself, in that the mean differences are insignificant, then the pursuit of discriminant models is a waste of time, for unless they are correctly *specified* there will be a complete distortion of the true or underlying relationships in the model.

MULTIVARIATE LINEAR DISCRIMINANT ANALYSIS - THE TECHNIQUE

Multivariate linear discriminant analysis provides an optimal linear transformation of a set of variables which are usually, but not necessarily, called ratios in the case of distress prediction/classification. Where the means of the ratios of the groups of failing and non-failing companies are multivariately normally distributed the z-scores will be univariately normally distributed. Whether or not the data actually does conform to the multivariate normal distribution is another question. If the respective group means of these z-scores are well separated on the number line, the researcher has a good chance of discriminating between those companies that are failing and those that are not. The extent to which this separation is adequate or otherwise, is a function of the significance of the difference between the respective mean vectors of the original ratio data, and the intercorrelations between those individual ratios or other variables.

Where the variance - covariance matrices are unequal however, the recommended approach is to use quadratic discriminant analysis. Comparisons between the results of linear and quadratic discriminant analysis have been carried out, [Lachenbruch, 1975]. In some cases the results of the linear model have been just as good a predictor or classifier as those of the quadratic model. This tends to reinforce the notion that the matter of the equality of the respective variance - covariance matrices is of little importance. Part of this research attempts to explain why linear discriminant models sometimes fail and why they are sometimes successful despite the breach of the equality assumption.

Linear discriminant computing programmes usually assume that the underlying population variance - covariance matrices are equal. When the sample data does not reflect this phenomenon an estimate of the so-called population matrix is made. The normal computing procedure is to take a pooled estimate of the variance - covariance matrices based upon a weighted average of the respective sample sizes for failed and non-failed groups. While this approach may seem economical in some ways, it creates problems because there is an interaction effect when combined with *a priori* probabilities.

It is important to remind ourselves here also, that the objective of sample based research is to produce statistical models which estimate population parameters in an unbiased manner. Criticisms of the sampling strategies employed by researchers working in the field of corporate distress classification/prediction are well documented, [e.g. Jones, 1987]. What is particularly important to recognise as far as this paper is concerned, is that in computing the sample discriminant function and the associated z-scores, we should be attempting to obtain the best estimates of the population discriminant function and associated z-scores for the particular population of companies which we are investigating. Furthermore, when we use such samples we should be attempting to estimate the population distributions of these z-scores for both failed and non-failed companies. This point is almost invariably over-looked in the literature on distress modelling. That this may lead to a biased estimate of the cut-off point has been dealt with, [Overall & Klett, 1972], what is not so widely appreciated, is that the objective should be to estimate the distribution of z-scores for the respective populations of failed and non-failed companies, because in subsequent applications of any model that might be developed, we will select a company and evaluate the extent to which it comes from the population of failed or non-failed firms.

TYPE I and TYPE II ERRORS.

Any statistical modelling process involves the risk of misclassifying the observed data and thereby developing inappropriate conclusions. When we attempt to develop a classification or prediction model that helps us determine whether a firm is likely to fall into the bankrupt of non-bankrupt category, we are in danger of inaccurately classifying at least a proportion of the time. There are two types of errors, Types I and II, that we can make in this respect. Altman [1968] defines a Type I error as the misclassification of a bankrupt firm as being non-bankrupt, and conversely, a Type II error, as that of misclassifying a non-bankrupt firm as being bankrupt. The size of our Type I and II errors provides us with an indication of the usefulness of our models. *A priori* probabilities affect the size of our Types I and II errors however.

A LITERATURE REVIEW:

Although much has been written on the subject of multiple linear discriminant analysis since it was first developed by Fisher in the 1930s, it was of course, not until Altman first published his research on predicting corporate failure in 1968 that the question of the application of the technique in this field became a matter of academic debate. Joy & Tollefson [1975] provided the first methodological critique of the specific use of the technique in corporate classification and prediction. Eisenbeis [1977] contributed to the issue very significantly with his paper in the *Journal of Finance*, and followed this with a joint publication on the matter a few years later, [1981]. Several other significantly important papers on the subject were published in the 1980s; Zmijewski [1984], Jones [1987] and finally a working paper by Pacey & Pham [1988]. Each is reviewed in order to outline their particular contribution to understanding why discriminant models of corporate failure ultimately fail themselves.

Joy & Tollefson outline the role of multivariate discriminant analysis in their paper. *"If the m attribute measurements arise from multivariate normal populations such that the categories have identical variance - covariance matrices, but different mean values for the attributes, then linear multiple discriminant analysis provides an optimal solution to the classification problem. When the measurements arise from multivariate normal populations, but the variance - covariance matrices are not identical, quadratic rather than linear multiple discriminant analysis yields the optimal solution,"* [p.723]. They point out that the majority of studies do not appear to use tests to establish whether optimal solutions are available with the particular data. Applied researchers all too frequently do not statistically evaluate the extent to which the ratios of failed and non-failed companies have equal variance - covariance matrices. Models developed in this manner, they argue, are less than optimal.

Joy & Tollefson are also critical of the fact that the sampling frames for the populations used to develop models are frequently unsatisfactory. In using the example of discriminant models which supposedly distinguish between good and bad loan applicants, they point out that examples of those who have been denied credit are frequently excluded. *"Sample sizes must also be chosen with recognition that two populations are sampled,"* [p.725]. Although data shortages are frequently a major problem for those researching into corporate failure, particularly outside the United States of America, it is essential that proper attention be paid to the sampling issues. Failure to select random samples will merely result in models that will be highly unlikely to generalise. They will, at best, be sample specific.

Thirdly Joy & Tollefson take care to point out the difference between *ex post* and *ex ante* discrimination. The former is achieved when we validate a model on a *hold out* sample taken from the same time period. *Ex ante* prediction involves forecasting within some future time period. The earlier discriminant models failed to recognise this distinction and provided no valid basis for forecasting. Intertemporal, or *ex ante* validation is a necessary condition of the development of forecasting models of corporate distress.

The same authors also point out that there were misleading interpretations of two important aspects of Altman's research. Firstly the relative importance of individual variables in Altman's work was incorrectly judged by standardising the discriminant coefficients by their respective standard errors. They refer readers to the work by Mosteller & Wallace, [1963] who developed the correct methodologies. Secondly, they examine the methods of evaluating the classification efficiency in detail and make suggestions about better methodologies here too. The importance of *a priori* probabilities, and the costs of misclassification are discussed in detail.

Eisenbeis, as a research specialist, had written extensively in the field before making his important contribution to the arguments about the methodologies involved in the application of multiple discriminant analysis to corporate distress, [1977]. He discusses several aspects of failure in this critique.

The distribution of the variables is his first concern. Violations of the multivariate normality assumption *"may bias the tests of significance and estimated error rates,"* [p.875]. He surveys the literature on investigations into deviations from the multivariate normal distribution including that of the use of dichotomous variables. In summarising the work of Lachenbruch, Sneering, and Revo [1973] Eisenbeis points out that *"the authors concluded that the standard linear procedures may be quite sensitive to nonmultivariate normality,"* [p.876]. He also points out that *"they also observed that even attempts to adjust for inequalities of the group dispersions by using quadratic classification rules did not significantly improve the results, and in many cases were worse,"* [p.877]. Finally he cautions against the unquestioned assumption that transformations of the marginal distributions of the ratios to normality on the grounds that this will *"not necessarily make the joint distribution more normal,"* [p.877], and adds that such transformations *"may change the interrelationships among the variables and may also affect the relative positions of the observations in the group,"* [p.877]. There is no easy solution to the question of the extent to which the violation of the multivariate normal distribution assumption will invalidate the results of MDA models of corporate distress. Pacey and Pham, [1988] assure us that the last word has not been written on this matter. *"The requirement of multivariate normality of discriminating variables seems critical to the estimation procedure. It is an area for further research,"* [p.15]. In attempting to explain some aspects of why models of corporate failure models themselves fail, this thesis examines an important facet of this question by investigating the extent to which reasonable levels of classification accuracy may be achieved with ratios and other variables which are univariately but not multivariately distributed.

Eisenbeis also argues that *"a second critical assumption of classical linear discriminant analysis is that the group dispersion [variance - covariance] matrices are equal across all groups,"* [p.877]. Although he summarises the literature on the topic a little more comprehensively than Joy and Tollefson, he adds little new material that helps applied researchers analyse the data that they do have. It appears that most ratio data from failed and non-failed companies do not have equal variance - covariance matrices.

In discussing the question of the interpretation of the significance of individual variables Eisenbeis assures us that *"one of the most widely misunderstood aspects of discriminant analysis relates to the problem of determining the relative importance of individual variables. Unlike the coefficients in the classical linear regression model, the discriminant function coefficients are not unique; only their ratios are,"* [p.883]. The scene is very confusing to say the least. Eisenbeis, Gilbert & Avery [1973] investigated six methods, [they rejected suggestions by Mosteller & Wallace and recommended by Joy & Tollefson], and concluded *"that all the methods for investigating the relative importance of variables that had currently been examined have assumed equal dispersions,"* [p.885]. This clearly leaves the applied researcher in the dark with respect to how to interpret the relative contribution of the variables in the model.

Of primary concern to applied researchers with small samples and a large number of potential or actual variables, is the reducing of the *dimensionality* as it is frequently called. Eisenbeis spends some time on this topic. *"Eisenbeis and Avery [1973] have examined in an heuristic manner the* *relationship between the significance tests for the equality of group means and the problem of investigating group overlap through classification methods. They argue that the existence of statistically significant differences among group means, especially when the sample sizes are large, does not convey much if any useful information about the ability to construct a successful classification scheme,"* [p.885]. He is critical of the attempts to reduce the number of variables because not only is it difficult to measure the contribution or significance of individual variables, but also because it is difficult to obtain objective measures of classification success or otherwise.

As Eisenbeis notes, *"discriminant analysis procedures assume that the groups being investigated are discrete and identifiable,"* [p.887]. Joy & Tollefson also point out that defining the two groups in the case of failed and non-failed companies may be a very difficult problem. Eisenbeis argues that there is a major problem in defining groups when a continuous variable is used as the criterion for classification. The cut-off point is usually arbitrarily defined. In the case of corporate distress classification and prediction the discussion should be extended. If corporate collapse is viewed as being on a time continuum, and in the case of some companies it is only a matter of time before they collapse, then we might question their definition as non-failed companies. They may or may not have all of the characteristics of a failed company, but be wrongly included in the non-failed group. This problem is overcome if sufficient time is allowed before the study is undertaken.

A priori probabilities are also discussed at some length by Eisenbeis. These probabilities do not influence the determination of the discriminant coefficients but they do have a marked influence on the cut-off points, [Overall & Klett, 1972]. If the sample proportions are used to set the probabilities unsatisfactory or biased cut-off points will usually be obtained, particularly as there has been a strong tendency for researchers to use equal sample sizes in by far the majority of published studies. If population proportions are used then we might suppose that this would provide better estimates. The problem with this approach is that the sample data is frequently obtained from a period spanning more than 20 years. During this time the proportion of company failures may vary considerably from year to year as economic fortunes fluctuate. Pacey and Pham, [1988], provide a very harsh conclusion with respect to *a priori* probabilities and the ability of MDA models to predict corporate collapses. In short they say that multiple discriminant models *"cannot outperform a naive model which assumes all firms are non-bankrupt. The MDA models perform slightly better [than the naive model], but only with respect to the period from which the parameters are estimated,"* [p.15]. The possible reasons for this phenomenon will be discussed in chapter two.

Finally, Eisenbeis briefly summarizes the difficulties in estimating errors of classification. Those using the original sample method are *"consistent but biased"*, [p.894]. Those using the *hold-out* sample method are unbiased *"but require large samples,"* [p.984]. Those using the population method are probably the best, yet the appropriate population parameters, that is model specification, might be difficult to establish.

Zmijewski [1984] is primarily concerned with the biases that result from prediction models developed from non-random samples. *"Estimating models on such samples can result in biased parameter and probability estimates if appropriate estimation techniques are not used,"* [p.59]. The first point that he makes is researchers oversample distressed firms and produce what he calls choice-based sample biases. Secondly, Zmijewski argues that because applied researchers require certain types of information in order to construct discriminant models, those companies from which the particular data is unavailable are automatically excluded from the sample. *"If the probability of distress given complete data is significantly different from the probability of distress given incomplete data, the estimated model will be biased,"* [p.74]. The problem relates specifically to that raised earlier by Joy & Tollefson.

Jones [1987] provides something of an updated overview of the research methodologies based upon the work of Scott, [1981]. His survey includes issues which have already been raised in this paper. Jones points out that *"the more sophisticated models have been based on statistical or mathematical literature and have not provided economic guide-lines to aid in independent variable selection....without an economic understanding of bankruptcy, it will be difficult to ascertain whether a model developed from data from one set of companies is appropriate for predicting the bankruptcy of a company operating in a different economic or temporal setting"*, [p.135]. He also cites Foster [1986] and Rose, Andrews and Giroux, [1982] who provide studies into the macroeconomic aspects behind the incidence of bankruptcy. Although the matter is at least raised by these researchers their models do not attempt to integrate the microeconomic aspects with the macroeconomic aspects of corporate failure simultaneously within discriminant models. This thesis attempts to provide a rationale and a method for doing so.

In arguing for the adjustment of historical data for *"general or specific price-level changes"*, Jones raises an interesting question. Of course ratio data is unlikely to be affected by price-level changes unless specific price-levels impact differently upon various aspects of the financial reports. Norton and Smith [1979] confirm this.

In discussing the question of reducing the variable set, Jones discusses the employment of factor analysis to reduce multicollinearity as an alternative to the more commonly used stepwise discriminant technique. Eisenbeis [1977] argues against variable reduction and in doing so implies that, with discriminant analysis, multicollinearity is not a major problem, *"concern for dimension reduction should follow and not precede the development and validation of alternative classification schemes as has been the case in most of the applied literature,"* [p.887].

Jones cites various studies which have explored the use of factor analysis and notes that many *"have identified the same factors and that the factors appear to be stable over many years,"* [p.142]. Jones is hopeful that the employment of factor analysis will be fruitful. I am conscious of the pitfalls involved. Such a technique is based upon the correlation matrix. The first problem with the method is that in small samples, and small sample work is by far the more commonly reported research, it is possible that spurious correlations are obtained because of one extreme variable value that may attenuate the statistic. If this is the case then the factor analysis will provide misleading results which it will be unlikely to replicate. Secondly, in the classification or prediction of corporate failure, the main factors, which have been shown to be stable, such as those found by Pinches, Mingo & Caruthers, [1973], might not be the variables which indicate impending collapse. An insignificant variable might not correlate with any other variable and therefore in a sense be excluded from the main factor loadings. Yet this uncorrelated ratio might provide the key to forecasting collapse. Factor analysis, particularly where it identifies stable factors, may provide interesting results for some other types of research but it will not necessarily solve our problems in the early identification corporate collapse. It may be the ratios which are not stable, or the ones that do not track with other ratios which provide us with the ability to classify or predict impending company failure adequately.

Finally, Robertson and Mills [1988] deliver a scathing criticism of current practices in the use of MDA models in a wide ranging survey of the previous two decades of published research. They are highly critical of the reliance on traditional accounting ratios. *"They are sadly out of date ....* [and they argue that] *.... if the traditional ratios did reveal the 'true trends', why then do so many companies fail, or decline? Why do so many companies linger on in a period of decline before taking action?"* [p.71].

They emphasize the need *"to observe strict mathematical standards,"* [p.72], when using MDA modelling techniques and furthermore argue that *"the methodology assumes that if a model is developed for a particular industry .... [and] .... in practice it is difficult to know whether a company does meet a specific industry standard,"* [p.72]. In many cases the concept of an industry is so illusive that heterogeneity rather than homogeneity is the dominant feature. The problem is even more acute in small economies where at the most there might be only two or three companies in a particular industry. Robertson and Mills also take Altman to task for his so called manufacturing industry study when almost 10 companies out of his small sample of 42 cannot be said to belong to that group.

They are also critical of the manipulation of the *cut-off* point in MDA analysis. *"The cut-off point is not negotiable,"* [p.73]. It is determined mathematically and is optimal for a particular set of data. While I can see some justification for what they say , I am unsure of how to judge the validity of their argument. It is probable that more research is needed, or perhaps more rational argument is needed with respect to this point. Their point should, I think, be taken as cautionary at least.

The same authors also cite research in which subsequent investigators have changed or modified the definition of one of the ratios in order to simplify the MDA model. *"But the ratios cannot be regarded individually. Any change to a single ratio has repercussions in the whole model, including the ratios contained in the model, the weights and the cut-off,"* [p.73]. This aspect is explored in detail in this thesis as it is of major importance to the continued success or otherwise of MDA model of distress.

The second part of their paper is devoted to a explanation of their new methodology for financial ratio analysis. They argue that the emphasis on multivariate models has resulted in *"shift away from developing and understanding financial ratios ....* [and that] *.... the new methodology puts forward a framework to allow researchers to return to the business of developing financial analysis techniques which can be used by businessmen,"* [p.74]. They argue for a three dimensional analysis of ratios. The first is an examination of various alternative ways by which similar ratios can be calculated in order to identify the ones which have the greatest mean differences between the failed and non-failed companies. Secondly, they examine each of these in order to estimate what they call *"a misclassification score,"* [p.75], and argue for the selection of ratios that have the lowest misclassification rates. Finally, they argue for the examination of the annual percentage change in each of the ratios.

There is certainly a need to develop a more satisfactory approach to ratio analysis, and indeed model specification. Robertson and Mills ideas do sound reasonable, some of which will be explored in this particular work, not so much because they are central to this thesis but because some of the conclusions of this thesis will reinforce the need to concentrate on establishing variables that have means that are markedly different for failed and non-failed companies. MDA modelling will not be successful unless we can do this.

They conclude with an important statement, *"that a methodology based upon discriminant analysis requires such high levels in application and interpretation that it has little chance of being applied correctly in practice,"* [p.76]. While this statement is fundamentally correct; it is demanding, it would seem that there might be some ways around some of the problems involved. Surely if we are to develop a satisfactory method of predicting the extent to which individual companies are likely to collapse, we will need to be rigourous. In working on this issue I have attempted to take our knowledge a little further down the pathway of being able to build better multivariate models. However much researchers may dislike this approach, the models will need to be multivariate, for like most situations in life, relationships are complex. To argue against the collapse on companies being a multivariate phenomenon is to argue for a univariate approach. This is even more simplistic than the search for a satisfactory multivariate model. I have not set out to find a panacea of all of the problems, for indeed there might well be none, but have selected a few issues for exploration. These issues are outlined in the next section.

THE MAIN CONCERNS

Despite the plethora of criticisms of the use of discriminant analysis in classifying or forecasting corporate distress there are some salient issues that seem to have been overlooked in the pursuit of more suitable statistical approaches. In this thesis I contend that while the use of multiple linear discriminant analysis is very exacting, it is still a very useful technique. Furthermore, despite the critical assumptions of multivariate normal distributions and the equality of the variance - covariance matrices we can bend the rules a little provided that we understand what is happening. We should understand these limitations quite clearly as to fail to do so leads to the producing of models of corporate failure that will themselves fail rapidly either over time, or even over subsequent application to sample companies from the original population in the same time period. There is no panacea of all ills in this matter, but all is not so dramatically lost as many critics might lead us to believe.

The question of *a priori* probabilities has largely been ignored in the discriminant modelling of corporate failure, [Deakin 1977]. Altman and Levalle [1981] mention it in their paper on Canadian business failure but sidestep the problem by implying that it is too difficult. This is unacceptable because these probabilities are frequently relevant to the development of the models. Altman does attempt to use *a priori* probabilities in his Loan and Savings model [1977] but reports no significant improvement in an already very successful model. Overall and Klett [1972], and Deakin [1977] discuss briefly, but nevertheless very graphically, the perils of ignoring the question. Statistical inferences based upon models which have ignored such *a priori* probabilities, however, may be very misleading, [Joy & Tollefson, 1975].

Of the two important assumptions behind the multiple linear discriminant model, that of the equivalence of the variance - covariance matrices for the failed and non-failed companies is sometimes very critical. It appears that most researchers in this field do not even evaluate the extent to which their data meets the assumptions of multivariate normality and the equivalence of the variance - covariance matrices, [Joy & Tolefson, 1975]. Some models show that even when using linear discriminant analysis, with unequal variance - covariance matrices, as opposed to the quadratic model, the predictive power is excellent. Chapter two explores the question of the interaction between *a priori* probabilities and unequal variance - covariance matrices and discusses why some models predict quite satisfactorily when *a priori* probabilities are ignored and why others do not.

Mean ratios change from time to time, either because of sampling error or because mean ratios from one time period may differ from those of another period. Because MDA techniques basically operate off mean differences, changing mean differences between the ratios of failed and non-failed companies may create particular difficulties. Such variations may result in a different distribution of z-scores and thus our interpretation of them will also need to change. In an attempt to explore the affect of such mean variable changes, chapter three investigates the interaction between minor changes in mean ratio differences, *a priori* probabilities and unequal variance - covariance matrices.

As already discussed, the assumption of multivariate normally distributed data seems to be important, however, chapter four uses many very large simulated ratio data bases to investigate the question of using multivariate linear discriminant modelling with unvariately normally distributed data. This was done in order to examine the extent to which the breach of the multivariate normality assumption reduces the ability to discriminate between failing and non-failing companies.

At first glance, the sample and "*hold-out*" sample method of evaluating models of corporate failure seems to be a very enticing strategy, particularly given the researcher's desire to replicate findings in a scientific manner. Despite its very wide-spread appeal, this approach is far less efficient than using the full sample to estimate the population discriminant coefficients. This issue is reported in chapter five.

Multivariate statistical models need to be correctly *specified*. If they are misspecified, a vast array of problems occur in the estimation of the parameters. For example, macroeconomic variables have not been used as an integral part of MDA models. The shortage of adequate data obliges many researchers to develop multivariate linear discriminant models using information collected over a number of years. Frequently problems are encountered when applying and evaluating models in another time period. These models have invariably been microeconomic models which at the very best have recognised that ratios are industry specific. The debt ratio for a finance company is expected to be markedly different from that of a manufacturing company. Researchers have attempted to circumvent this problem by carefully selecting homogeneous samples. The problem, however, is more critical than this. A debt ratio of 85%, for example, may be unacceptable in one economic climate, but indicate no impending problems in another. Deakin [1972] built models for different time periods and in the same paper pointed out that ratios, and therefore the discriminant coefficients themselves, vary over time. Some of the focus of the research needs to move to the macroeconomic level. King [1966] showed that the macroeconomic environment was important to share prices and although it is discussed briefly elsewhere in the literature [Foster 1986], this context has largely been ignored by researchers using the MDA approach to classifying and forecasting corporate failure. Although developing a comprehensive MDA model using macroeconomic variables lies outside the domain of this particular research, chapter six explores a method by which macroeconomic variables might be incorporated. It also briefly explores the need for lagged variables, distributed lags, and the need for indicators of change, such as first differences to be developed as part of the modelling process. More careful attention needs to be paid to correctly specifying MDA models as misspecification will usually fatally destroy any model.

The last chapter, chapter seven, summarises each of the issues in relation to their relative importance. The relative importance of the issues is divided into two section; those of critical importance, and those of lesser importance. Surprisingly, the issue of the extent to which the data in multivariately normally distributed and the extent to which the respective variance - covariance matrices are equal are judged to be of lesser importance. Model specification, followed by the need for random sampling, effective and efficient use of the sample data, the need for stable mean vectors, the need for significant mean differences, and the need for *ex ante* validation are held to be the most critical for model development.

Although the use of MDA models has declined in recent years the central theme of this research is that their use is by no means obsolete. The technique must be used with caution and in the full knowledge of the effect of any breach of the assumptions and other sampling issues. It is clear that its naive use will lead to a naive model which will not stand any test of time despite the fact that they might seem to fit a data set with a high degree of discriminatory power at the time.

Chapter Two

THE INTERACTION OF *A PRIORI* PROBABILITIES AND UNEQUAL VARIANCE - COVARIANCE MATRICES IN CORPORATE DISTRESS MODELLING.

INTRODUCTION:

This chapter addresses the use of *a priori* probabilities of group membership and the interaction of such probabilities with equal and unequal variance - covariance matrices as an important issue in MDA modelling of corporate distress. Despite wide spread debate over the matter, *a priori* probabilities of corporate failure are frequently disregarded in favour of an apparently more pragmatic approach. This philosophy is usually characterised by an expression such as, "Well it discriminates doesn't it? Why not just use it?". Sometimes ignoring *a priori* probabilities in developing MDA models appears create few problems, if any at all. This paper focuses upon the question of when and why *a priori* probabilities of company failure are able to be quite safely ignored. It also focuses upon the dangers of doing so. While the ignoring of *a priori* probabilities does not influence the estimates of the population discriminant coefficients when the respective variance - covariance matrices for failed and non-failed companies are equal, [Lachenbruch, 1975], [although this is not the case when MDA techniques are applied to unequal variance - covariance matrices], it does create a problem when cut-off points are selected. Other less formal techniques are usually employed, such as trial and error approaches to cut-off points, and the *a priori* probability method, the correct method, [Overall & Klett, 1972; Deakin,1977], is usually dropped. It usually dropped because, quite contrary to theory, it frequently appears to be irrelevant. Many empirical models seem to indicate that *a priori* probabilities are unimportant. In accepting this argument researchers have overlooked two important issues. Firstly, they have not explained why *a priori* probabilities are irrelevant, and secondly, and more importantly, I believe, they tend to have overlooked the main objective of the research that they are undertaking. They tend to fail to recognise that any statistical research uses samples in order to estimate the population parameters. The population distributions of the z-scores of failed and non-failed companies are of vital importance.

A PRIORI **PROBABILITIES OF FAILURE AND NON-FAILURE.**

A priori probabilities of group membership are defined as the probability that a randomly selected observation will belong to one of the groups under study. If, in the case of corporate distress modelling, a randomly selected company has a 5% chance of being a failed company, within say the next 12 months for example, it is because 5% of the companies in the total population will fail within that time period. Forecasting this figure with a reasonable degree of accuracy might seem be difficult as it will vary from year to year, but Rose, Andrews and Giroux [1982] have shown that this is able to be predicted on a quarterly basis in the United States.

A priori probabilities of group membership determines the positioning of the *cut-off* point on the z-score numberline, [Overall & Klett, 1972]. In the literature on corporate failure classification/prediction, the use of *a priori* probabilities is almost always neglected in favour of an arbitrary choice of a cut-off which seems to best fit the sample data. Robertson and Mills [1988] argue that in any particular data set, the cut-off point is determined mathematically and is therefore *not "negotiable."* Although the question of *a priori* probabilities has been addressed fairly thoroughly by the critics in more recent times, it was completely ignored in the early literature on corporate failure. Researchers using MDA techniques have been able to ignore this concept in some cases and still obtain satisfactory classification models, [e.g. Altman & Levalle, 1981]. Rising pressure in the literature has obliged researchers to confront the issue. The question of why some model builders have been able to ignore *a priori* probabilities and yet produce seemingly satisfactory models has not been adequately explained. This chapter attempts to do so.

VARIANCE - COVARIANCE EQUALITY

Not all company ratio data conforms to the ideal variance - covariance equality criterion. In fact, it appears that little, if any, of the data does. If we examine various conditions one at a time, *ceterus parubus*, we should be able to identify the circumstances under which we can reasonably depart from this particular formal assumption of MDA. We should then be able to identify the specific circumstances under which our attempts to classify or predict are more likely to succeed. A series of graphs developed from a series of simulations produces contrasting situations. These should enable us to identify the critical factor quite clearly. There are standardised three cases which require analysis; the equality of variance - covariance matrices for failed and non-failed companies, the situation where the variance - covariance matrix of failed companies is generally larger than that of non-failed companies, and finally, the case where the variance - covariance matrix of non-failed companies is generally greater than that of failed companies. Each case needs to be examined in turn as each provides us with a special set of circumstances. The situation in reality however, may be even more confusing as some variances and covariances may be larger than those of the other group while the opposite may be the case for other variables. The three extreme cases should allow us to observe the trends.

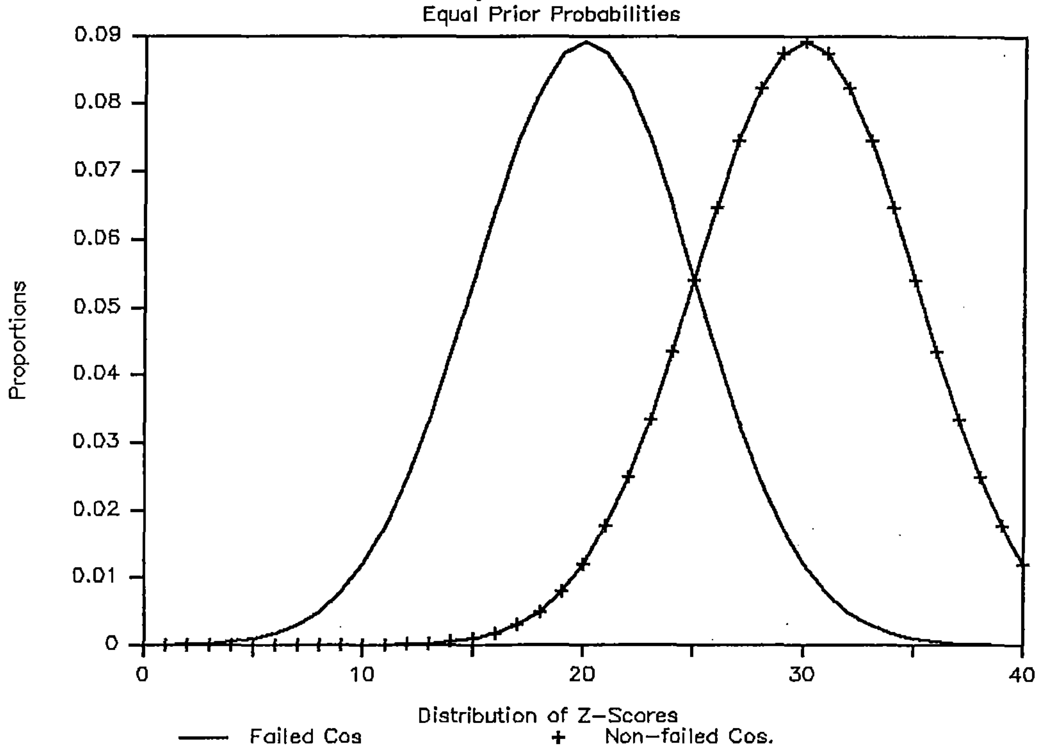
SITUATION I: LINEAR DISCRIMINATE ANALYSIS WITH EQUAL VARIANCE-COVARIANCE MATRICES.

If the variance - covariance matrices are equal the z-scores of both groups will have the same dispersion, i.e., the same standard deviation. [This matter is addressed more thoroughly in chapter three.] We can now examine two typical situations. The first, Case A, where the means vectors of ratios are moderately well separated and hence the centroids, [i.e., the respective group z-score means], of the groups will be moderately well separated also, and Case B, where the means are markedly well separated. In this latter example the centroids of both groups will be well separated in this case.

CASE A Means only moderately well separated.

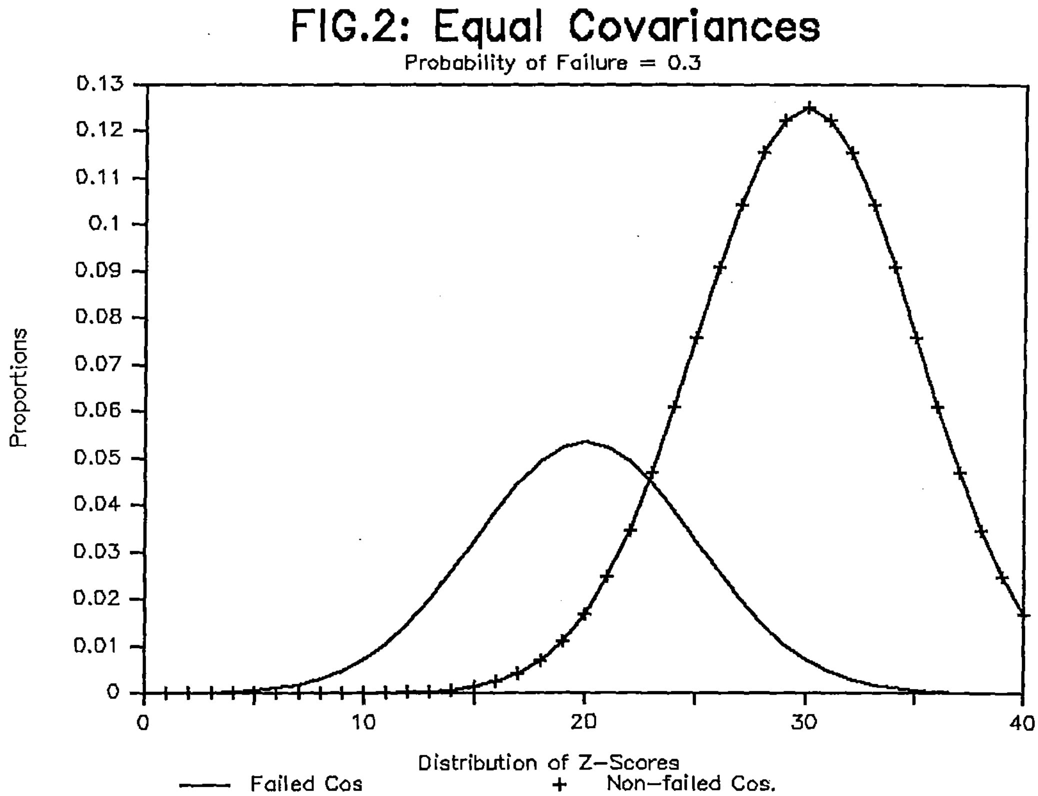
Even a cursory reading of the literature will show that the sample sizes of failed companies is usually matched, or fairly closely so, with those of the non-failed firms, [Altman, 1968, 1977; Deakin, 1972, 1977; Elam, 1975;Norton & Smith, 1979; et.al]. We can simulate this situation by graphing the data to show an hypothetical distribution of the sample z-scores for non-failing companies compared with those of failed companies on its left. The means, or group centroids, are different at 30 and 20 respectively, and the standard deviations are identical. Although there is clearly an area of overlap in the z-scores, the associated discriminant function discriminates moderately well at a cut-off score of 25. Figure 1 reflects this type of situation.

FIG.1: Equal Covariances



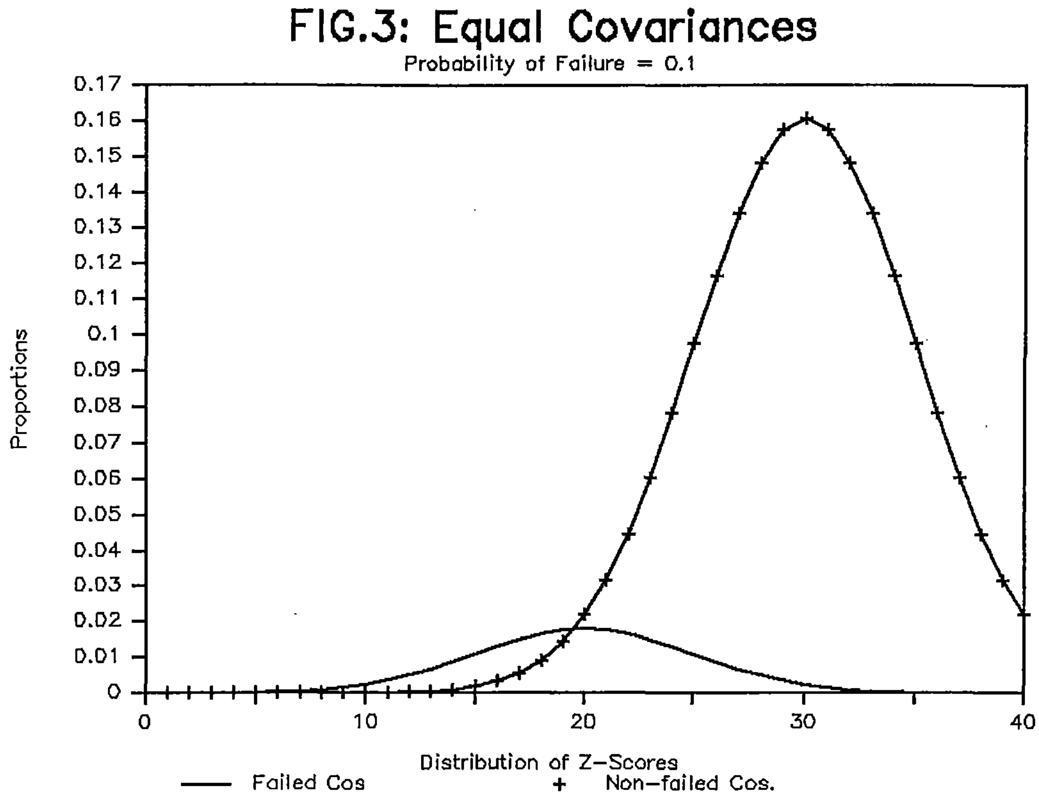
A company with a z-score of 20, for example, has a very high probability of belonging to the failed group of companies because the ordinate of the failed group at this point is much higher than that of the non-failed group. However, the equal sample sizes can easily mask the true situation.

If we now begin to adjust the sample data for the a *priori* probabilities, or population proportions, the trend begins to emerge. If we hold the means constant and assume that 30% of the company population are failing firms, the population distributions show that we begin to lose the ability to discriminate. If we now compare the first and second graphs we can see that a significantly larger proportion of the failed firms now have z-scores which are overlapped by the distribution of the z-scores of the non-failed firms.



Although still highly likely to belong to the failed group of companies, a company with a z-score of 20 is much less likely to belong to this group because the respective population ordinates change as adjustment is made for *a priori* probabilities.

International statistics reveal that far fewer than 30% of registered companies fail in any one year. The figure ranges between about one and eight percent, depending on how we define our population. If for the purposes of this discussion we assume a much more realistic *a priori* probability of about 10% for failed companies the situation is clearly displayed. Our population model of corporate failure based upon equal sample sizes now begins to fail quite dramatically. Type I errors are noticeably larger than Type II errors.



Now with a z-score of 20, a company is more likely to belong to the non-failed group.

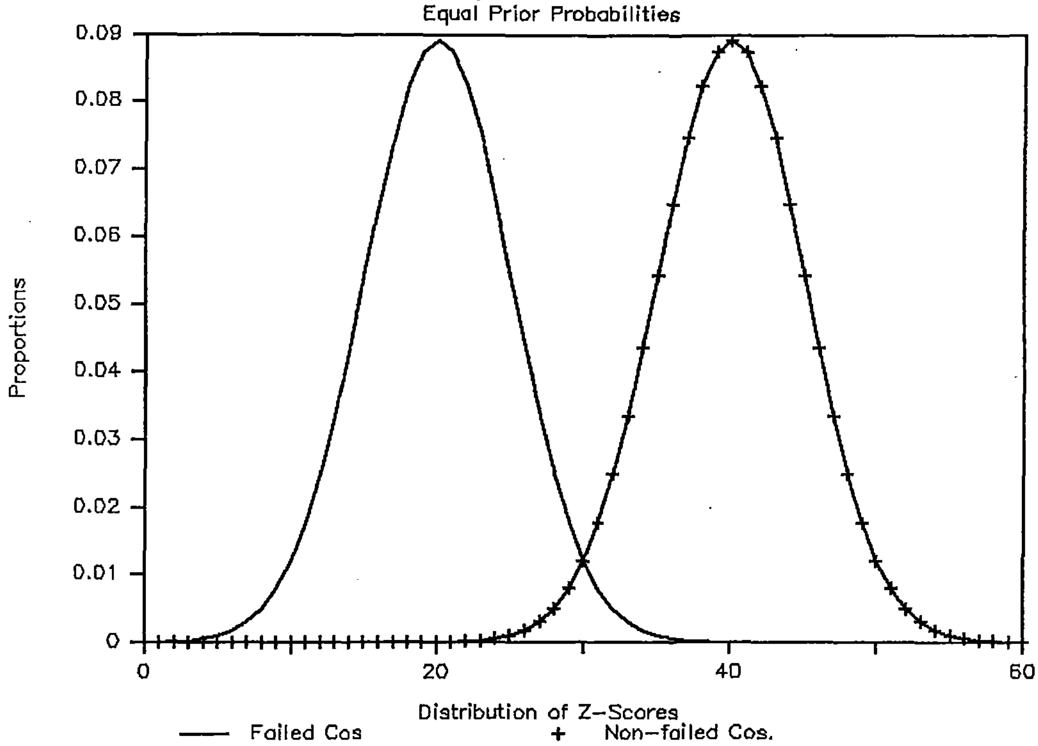
Despite our having what at first appears to be a reasonable model of corporate failure based upon the hypothetical sample data, what we are left with, once we have estimated the respective population distributions, is much less satisfactory. In other words, although we may have matched pairs in our sample, and although the statistics, such as the F ratio, the Lambda and the Mahalanobis D-squared, appear to be satisfactory, the failure to adjust for *a priori* probabilities, and the failure to attempt to produce some sort of estimate of the respective population distributions, leads to unacceptable errors. Type I errors increase even more dramatically. This is not so say that our initial model is entirely useless. It is a much better predictor of sound companies. This might be sufficient for our particular purposes, but it provides an unsatisfactory model for predicting/classifying the failed companies.

While these examples tend to point us in the direction of an answer to the question of why models of corporate failure fail to predict themselves, the situation is not necessarily a hopeless one. If we retain the assumptions of multivariate normality and the equivalence of the variance - covariance matrices, and examine the case where the centroids are well separated, we should gain further insights into why they may in fact succeed.

CASE B Means markedly well separated.

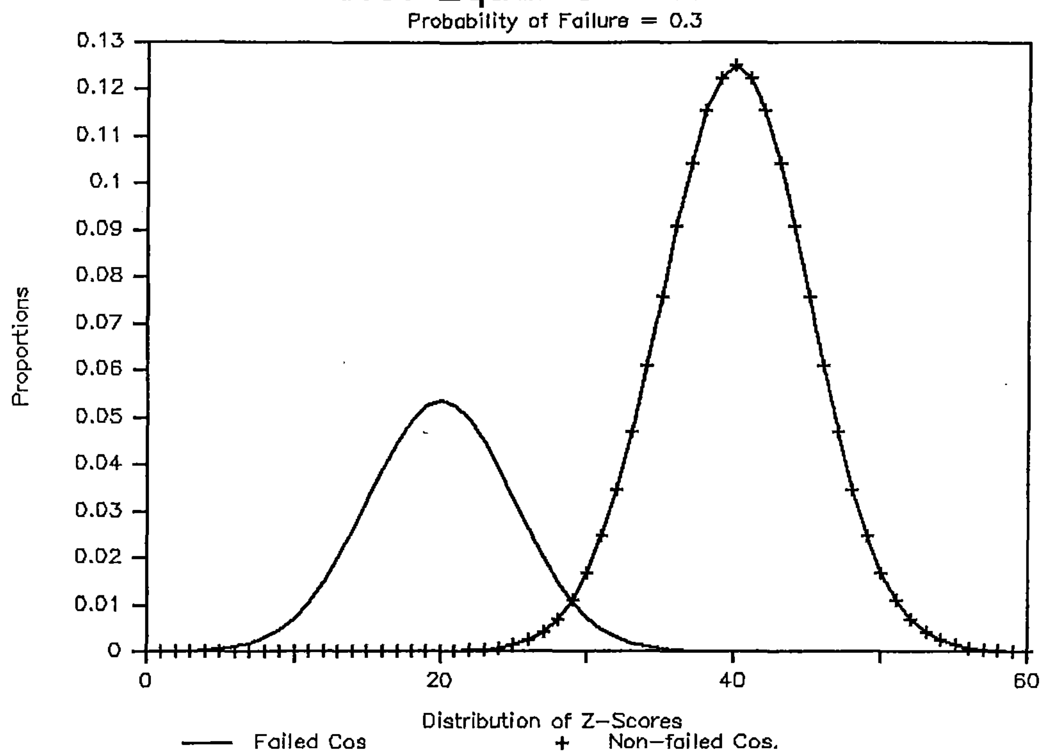
Where the mean vectors of ratios are markedly well separated there is clear discrimination at the sample level because the optimal linear transformation produces clusters of z-scores whose means are also markedly well separated. The simulated sample data shows this quite clearly when graphed. With the means or centroids at 20 and 40 respectively there is very little overlap. Misclassification is minimal.

FIG.4: Equal Covariances



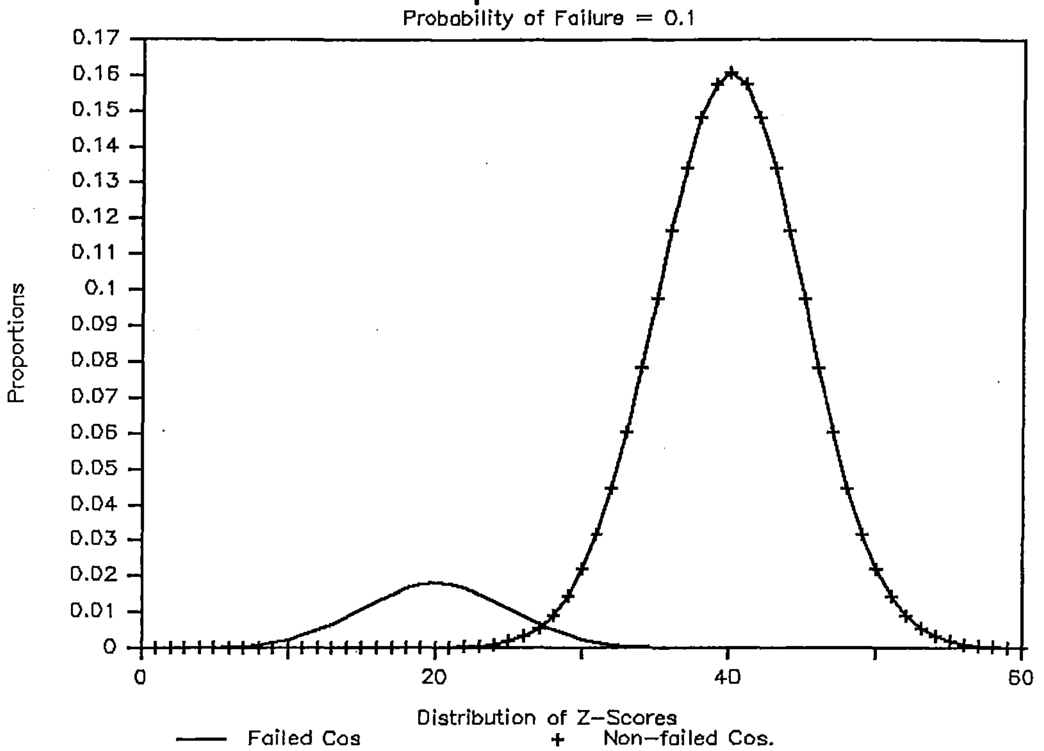
As we move through to a 30% *a priori* probability our ability to discriminate between the two possible outcomes is not lost or reduced. The failing companies stand out very clearly as having low z-scores while the zone of confusion, or *the zone of ignorance*, as Altman called it, remains very small indeed.

FIG.5: Equal Covariances



Finally, when we move to a failure rate of 10%, the discrimination in the population is almost perfect. The degree of overlap between the distributions is very small indeed. Clearly those companies with low z-scores are the failing companies.

FIG.6: Equal Covariances



In summary, the standard deviations of the z-scores are equal because the variance - covariance matrices are equal. The vectors of mean ratios are sufficiently well separated to allow excellent discrimination. The extent of discriminatory power is not severely restricted by the failure to use *a priori* probabilities, and both Type I and II errors remain relatively small and approximately equal.

So far we have assumed the equality of the variance - covariance matrices and thus equal dispersion for each group of z-scores. If we now relax this assumption, and this is much more frequently the case in empirical work, then the z-scores themselves will be distributed differently,[this aspect is the subject of more detailed analysis in chapter three]. While the general literature provides studies that use a quadratic discriminant model, (Eisenbeis, 1977), many researchers into corporate distress classification/prediction have shown that they can continue to use the MDA model quite successfully. What is the effect of this, and, to what extent can we discriminate under these conditions, despite the fact that we might ignore *a priori* probabilities?

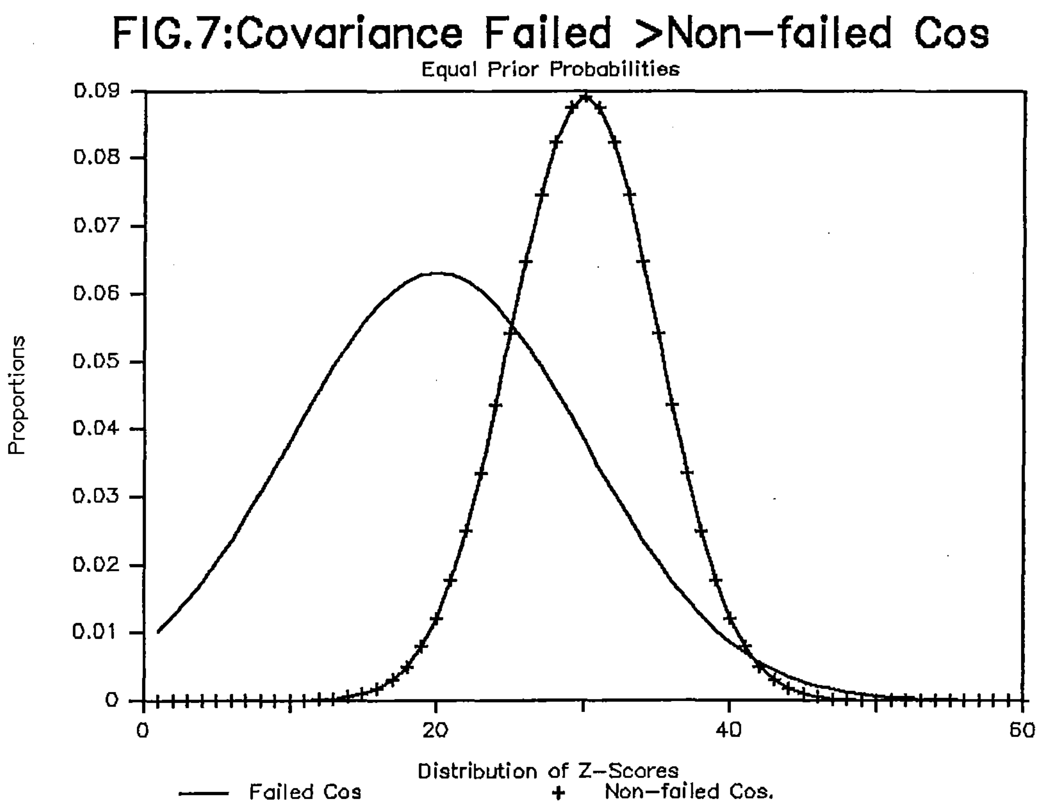
Again we need to examine two possible generalised cases. Firstly we will examine the situation where the variance-covariance matrix of the failed companies is larger than that of the non-failed companies. In this case then the standard deviation of the z-scores of the failed companies will be larger than that of the non-failed companies.

SITUATION II: VARIANCE-COVARIANCE MATRIX OF FAILED GROUP IS GREATER THAN THAT OF THE NON-FAILED GROUP.

CASE C - Means only moderately well separated.

As we did in case A where the variance-covariance matrices for both groups were equal, we commence with equal sample sizes and different means, except that we now assume that variance - covariance matrix of the failed companies is the larger and thus the dispersion of the z-scores for the failed companies will also be the larger. Although there are almost an infinite set of possibilities it will be sufficient for the purposes of this argument to follow through one hypothetical case.

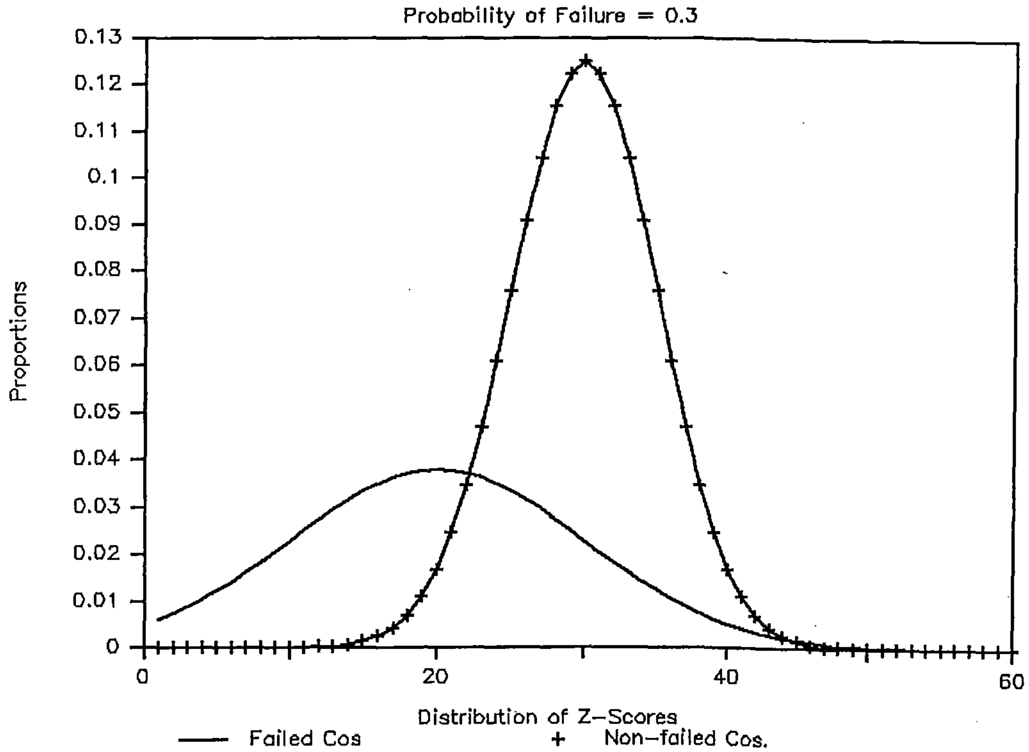
Commencing with equal sample sizes, multivariate normality, and different centriods, and with the mean vectors of ratios being significantly different, the sample distributions will appear as portrayed in figure 7.



The means are 20 and 30 respectively and the two sample distributions intersect at two points. Z-scores lying between these two intersection points are more likely to belong to companies which are likely to succeed, although there is a very large degree of overlap.

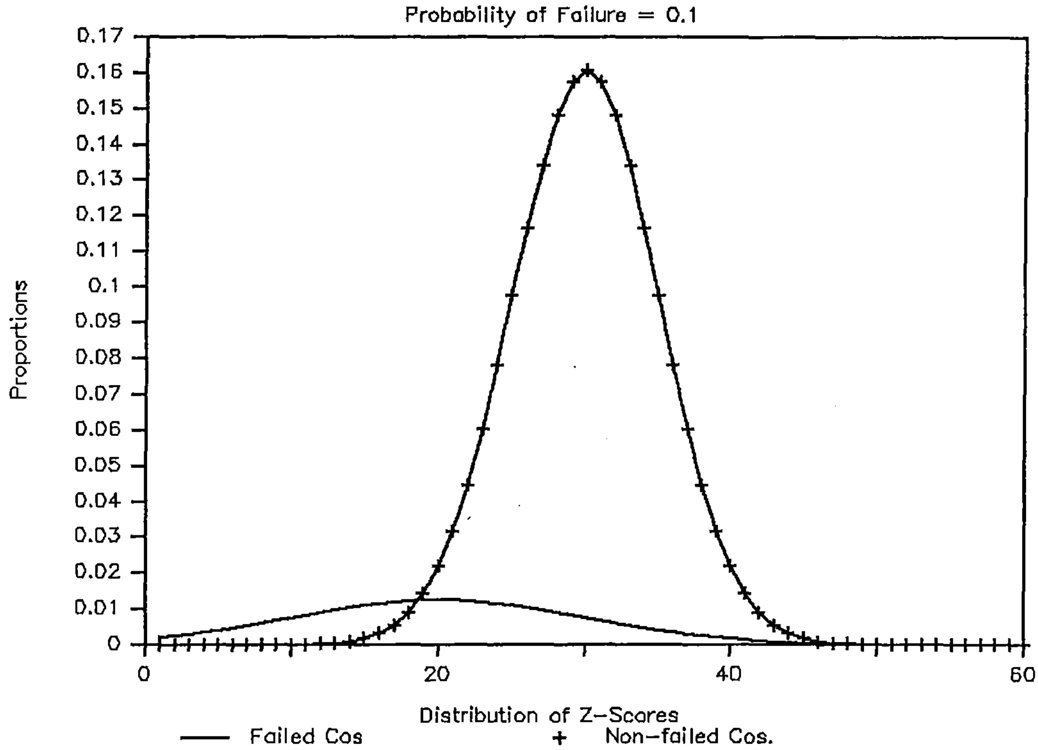
Now as we move towards a significantly lower proportion of failing companies in the total population, the population distributions of failed and non-failed companies will tend to merge as shown in Figure 8.

FIG.8:Covariance Failed >Non-failed Cos



Discrimination is still possible at both ends of the z-score scale, although it might not be deemed worthwhile at the lower end. Again as we move to a much more realistic 10% company failure rate the use of the model becomes much more tenuous.

FIG.9:Covariance Failed >Non-failed Cos

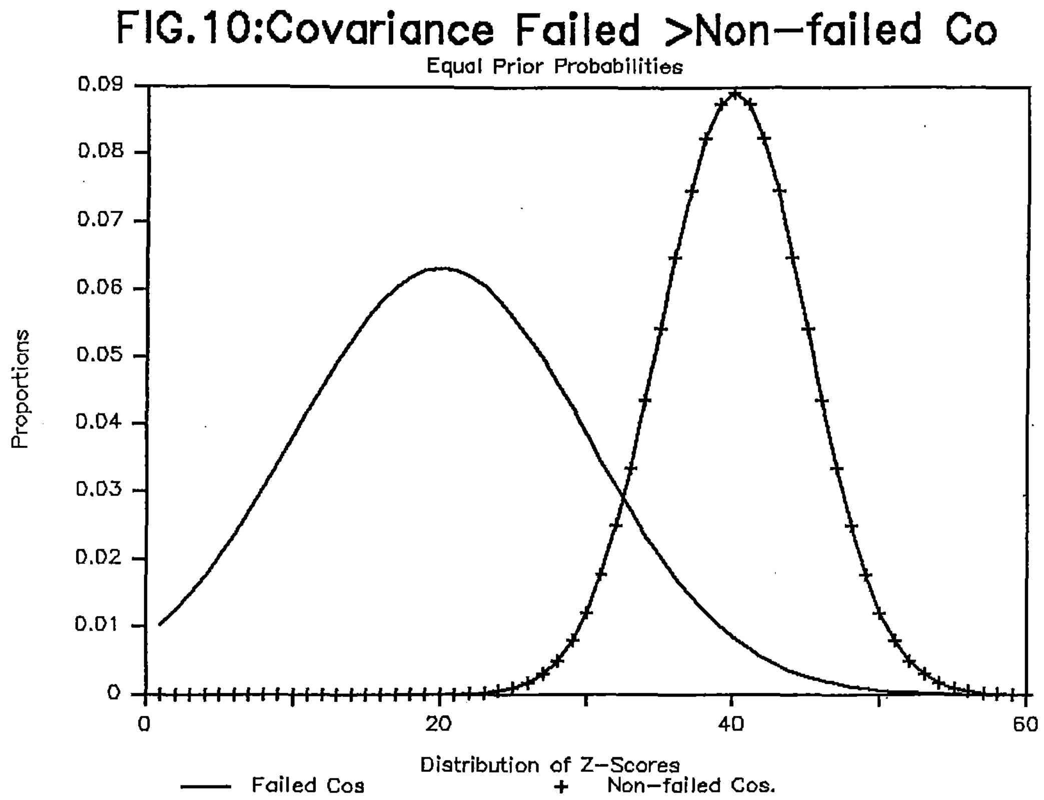


The final scenario shows that when the *a priori* probability of company failure is at the 10% level there is very little basis for discrimination at all. Although it might argued that the model is useful at the lower values, this would depend on the costs of misclassification. At the cut-off point the Type I errors are much larger than the Type II errors. The possibility of misclassifying a failed firm as a non-failed firm is much larger than the converse.

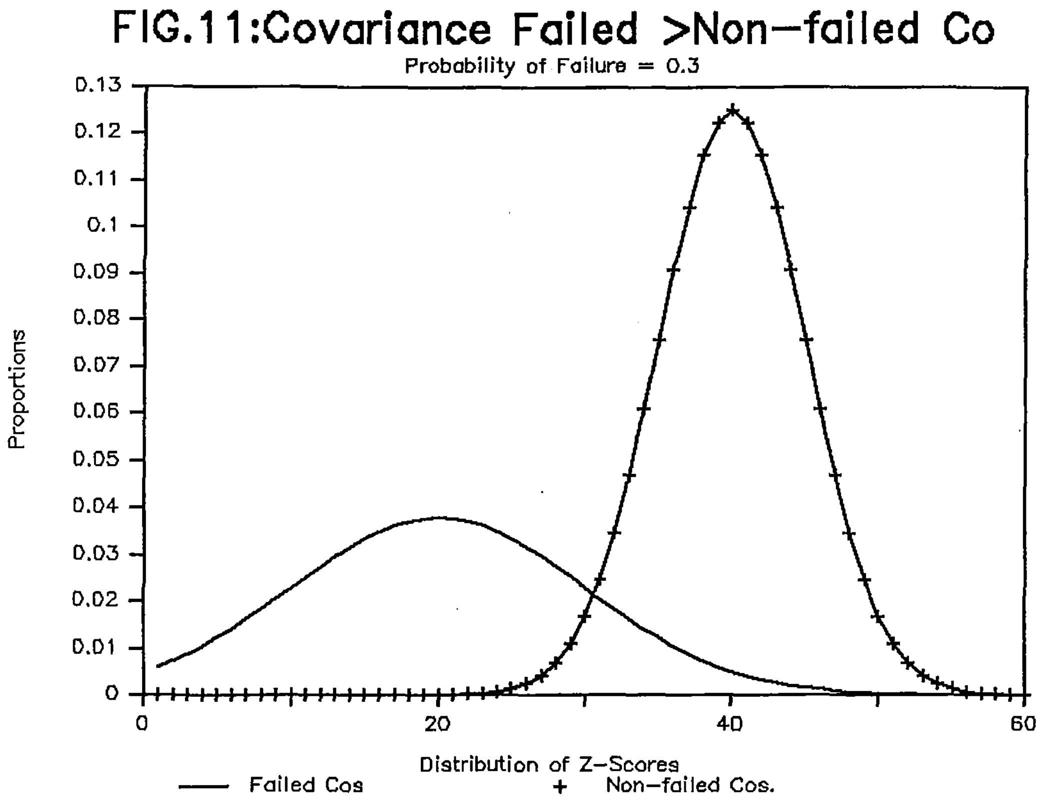
At the point of intersection, where the two distributions meet, the ordinates of both groups are equal. An examination of figures 7,8 & 9 shows that this point of intersection moves progressively towards the left as adjustment for *a priori* probabilities is made.

CASE D - Means well separated

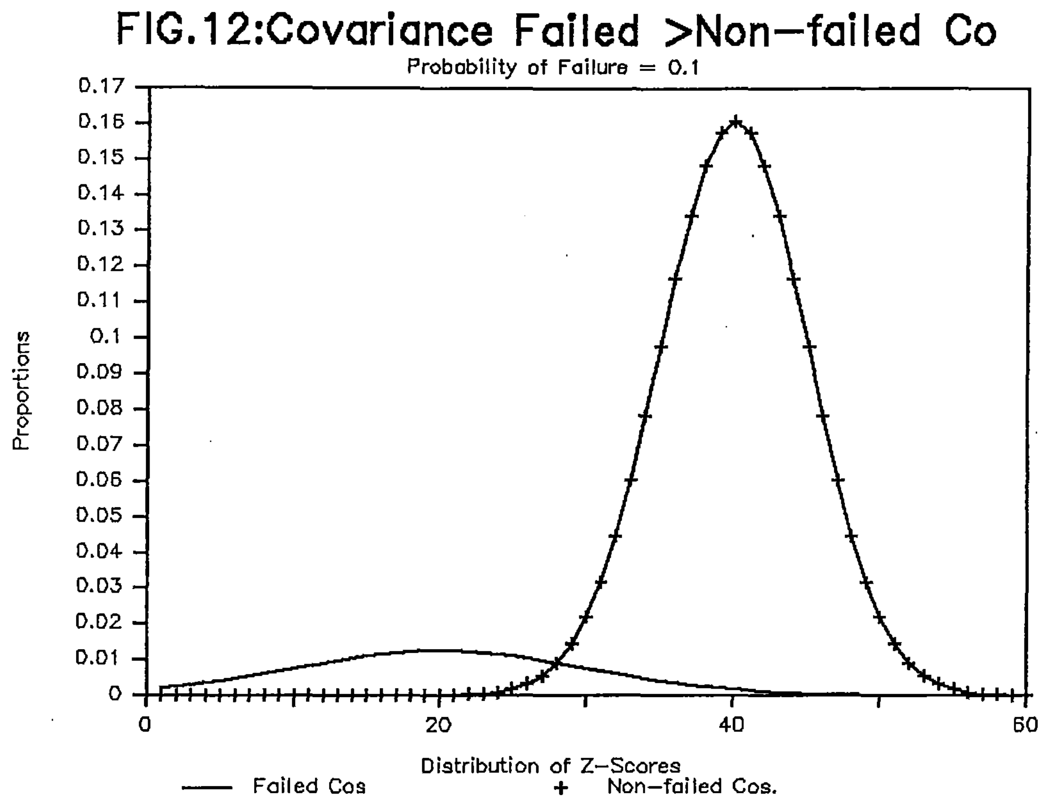
Now if we recommence with the same situation as in Case C, except with mean vectors of ratios which are well separated, the resultant centroids will also be well separated. With z-score means of 20 and 40 respectively the sample distributions show that a fairly sound basis of classification/prediction can be achieved. Type I errors remain larger than the Type II errors.



The next simulation shows that the model is fairly robust. As we move to a 30% rate of company failure the cut-off point has moved slightly to the left and is positioned at a z-score of approximately 30. If anything, there appears to be a slightly better ability to discriminate between the two groups.



At the 10% level of company failure in the population we still have clear discrimination, although perhaps not as dramatic as shown in the two previous graphs. Clearly, there is no lack of discriminatory power. Type I errors are still larger than Type II errors.



Where the mean vectors of ratios are not so well separated, the ability to discriminate is, as we well know, always limited. As the mean vectors of ratios separate, our ability to discriminate increases. Where the variance-covariance matrix for failed companies is larger than of non-failed companies, i.e., where the dispersion of the raw score ratios is greater, the dispersion of z-scores will also be greater for non-failed companies. This will result in more Type I errors than Type II errors as we will predict that relatively more failing companies to be non-failing in error. Compare figures 3 and 9, and 6 and 9, for example. In both cases where the mean vectors are well separated, the percentage of Type I errors diminishes. We now need to examine the situation where there is a greater dispersion amongst the ratios of the non-failed group of companies.

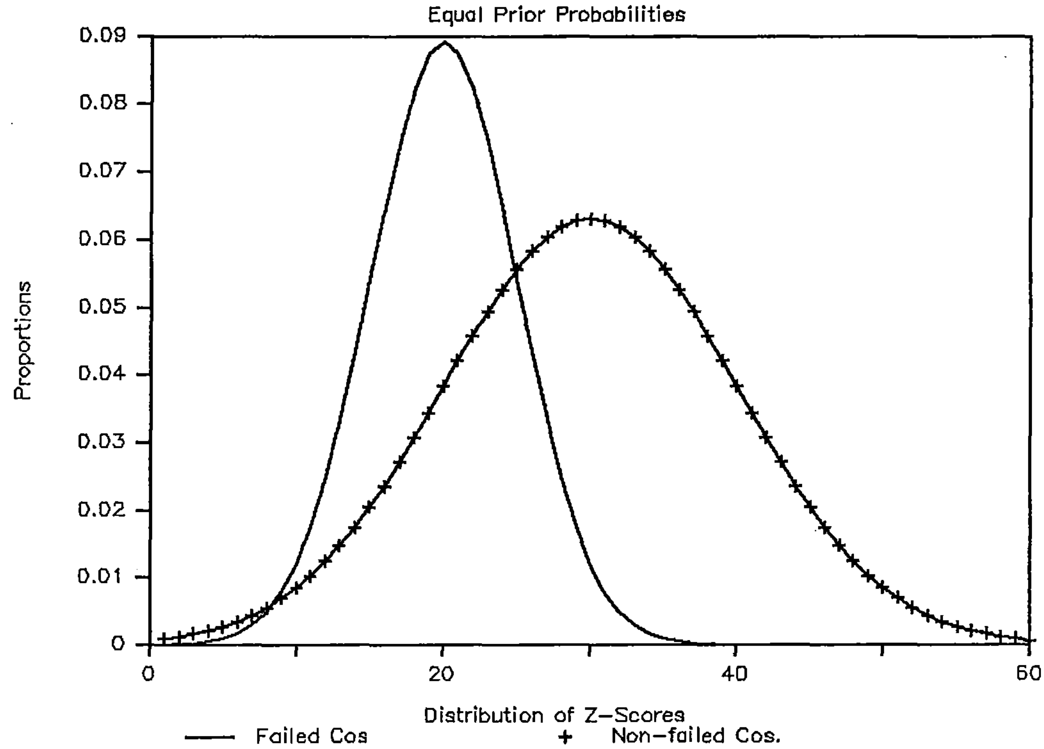
SITUATION III: VARIANCE - COVARIANCE MATRIX OF NON-FAILED GROUP IS GREATER THAN THAT OF THE FAILED GROUP.

Because the dispersion of the ratios amongst the non-failed companies is greater than that of the failed companies, the dispersion of the z-scores will also tend to be greater. This is the condition we are more likely to find in the real world. One of the problems with discriminant type research is that companies which do not appear to be failing frequently have a very wide dispersion of ratios. These when treated in a univariate sense provide a poor basis for prediction as on occasions, many of those non-failing companies have very poor ratios - yet do not fail. Unfortunately this problem is not always able to be solved by using multivariate statistical techniques.

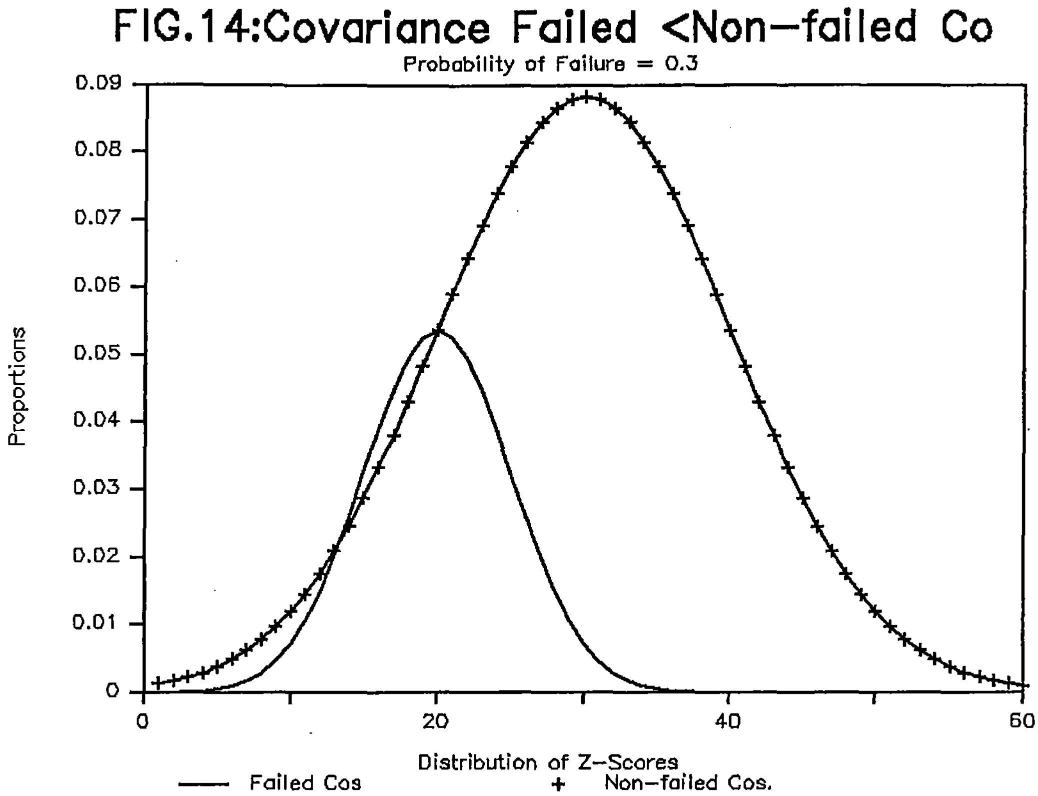
CASE E - Means only moderately well separated.

As we follow the same series of stages in cases C and D, figure 13 reflects the sample distributions. In this case however, Type II errors are significantly greater than Type I errors. That is, the probability of misclassifying non-failed companies as failed companies is much greater because of the wider dispersion of the non-failed company ratios, and hence the wider dispersion of the z-scores.

FIG.13:Covariance Failed <Non–failed Co

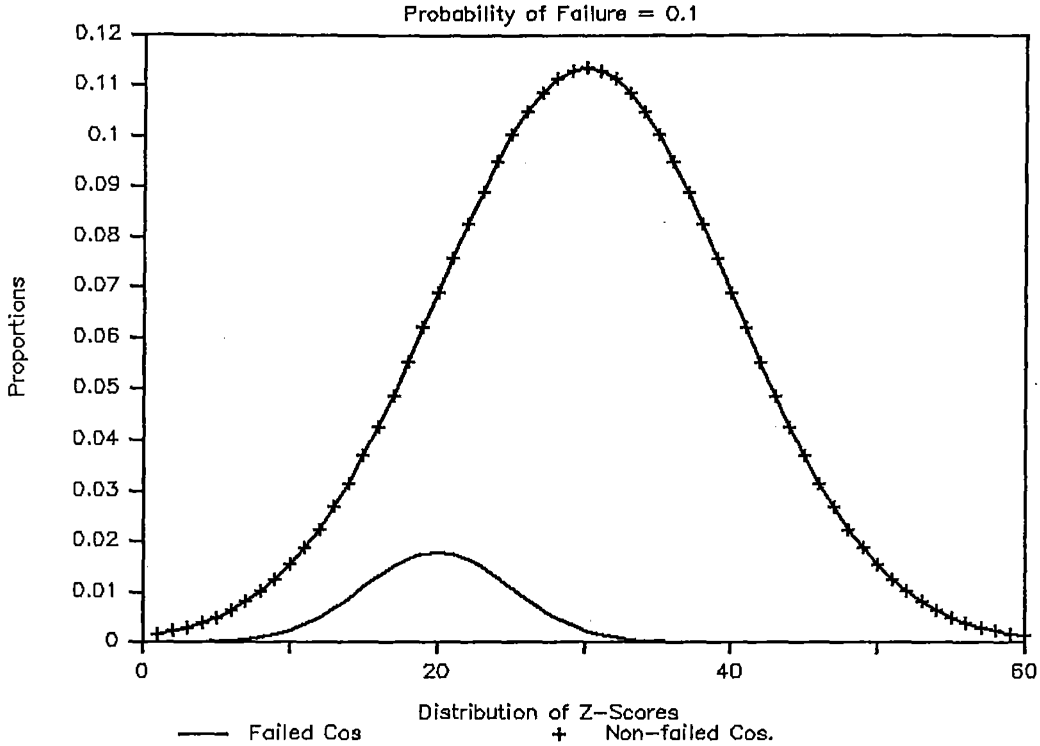


Only very modest discrimination is possible with a cut-off point of approximately 25. We may or may not regard our model as satisfactory depending on our specific objectives. If we now start to progressively estimate the respective population distributions by assuming an *a priori* probability of failure of 30% we have a markedly reduced ability to discriminate.



Finally, figure 15 shows that we have lost all discriminant power altogether. If we rely upon the model alone we will merely predict that all companies in the population will fall into the non-failing group. We do not need a discriminant model to demonstrate this feature. There is now a dramatic change in the Type I and Type II errors. The sample data showed a marked bias towards Type II errors. The population distribution reflects the converse. The sample data by itself is unable to correctly reflect types of errors. Type I errors now exist. Type II errors do not.

FIG.15:Covariance Failed <Non-failed Co

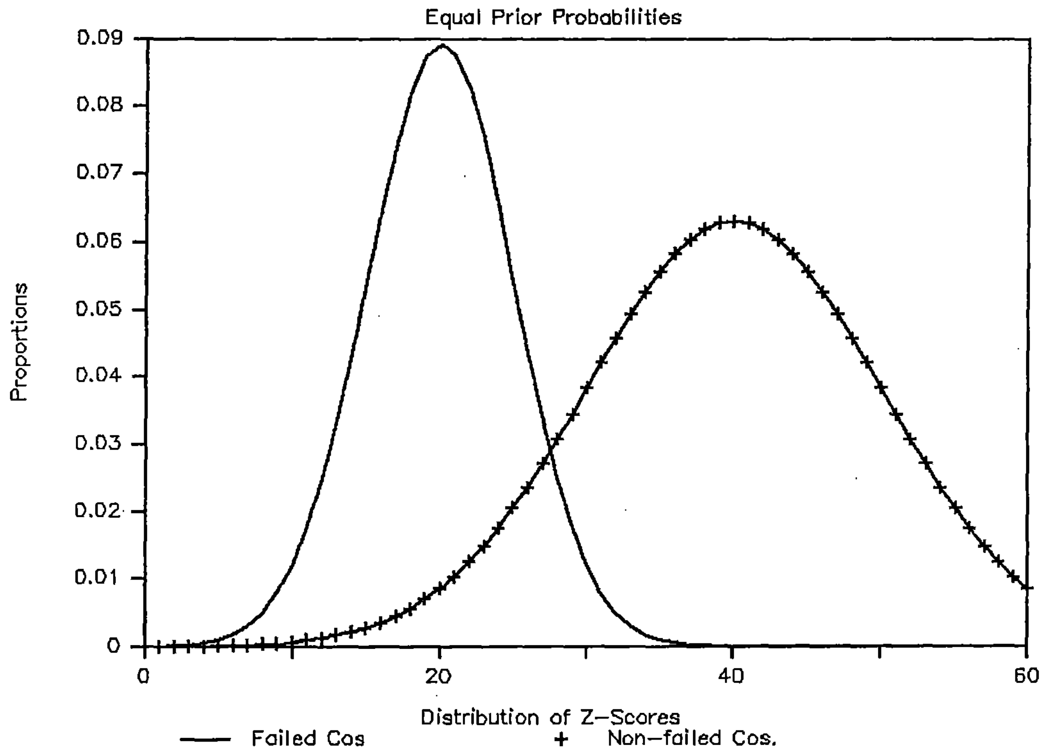


We need not restrict ourselves quite so harshly in reality. The models do identify a set of companies which require closer attention. This would require the identification of other discriminating variables as the ratios incorporated in the model provide only a partial discrimination. Clearly more information, i.e., a better model specification, possibly of a non-ratio type, is required before the model would discriminate.

CASE F - Means well separated

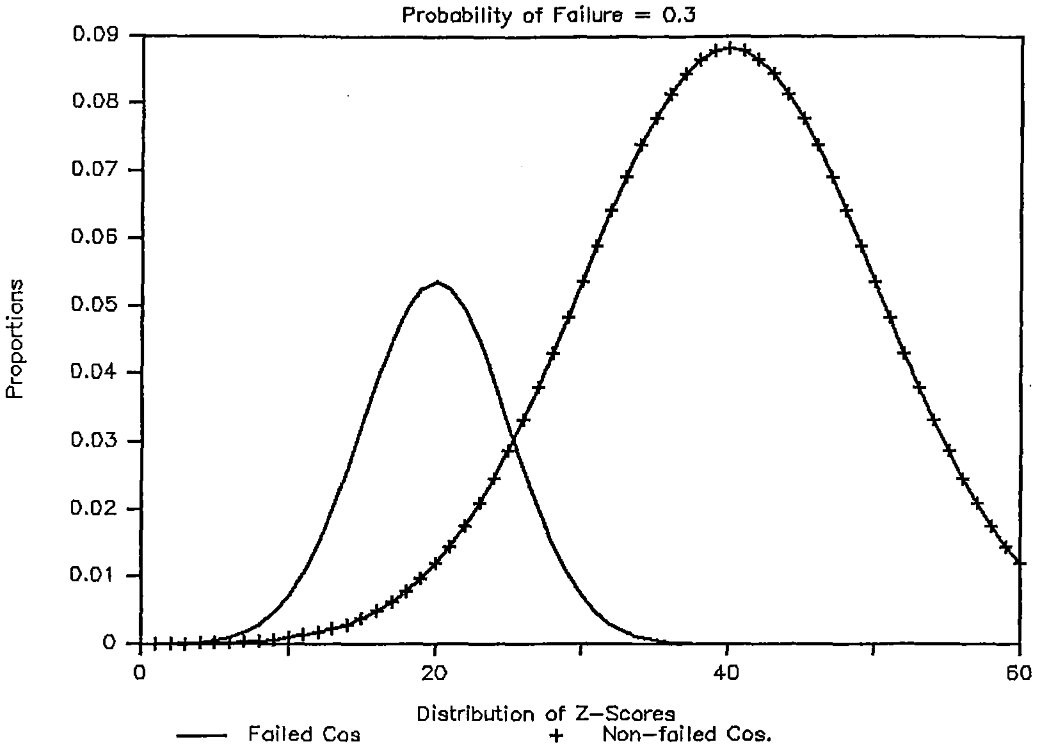
Our final set of scenarios provides for a much better distinction between the mean vectors of the ratios than Case E provided. Figure 16 shows the situation in which there is a fairly satisfactory discrimination in the sample data. Type II errors are significantly larger than those of a Type I nature.

FIG.16:Covariance Failed <Non-failed Co



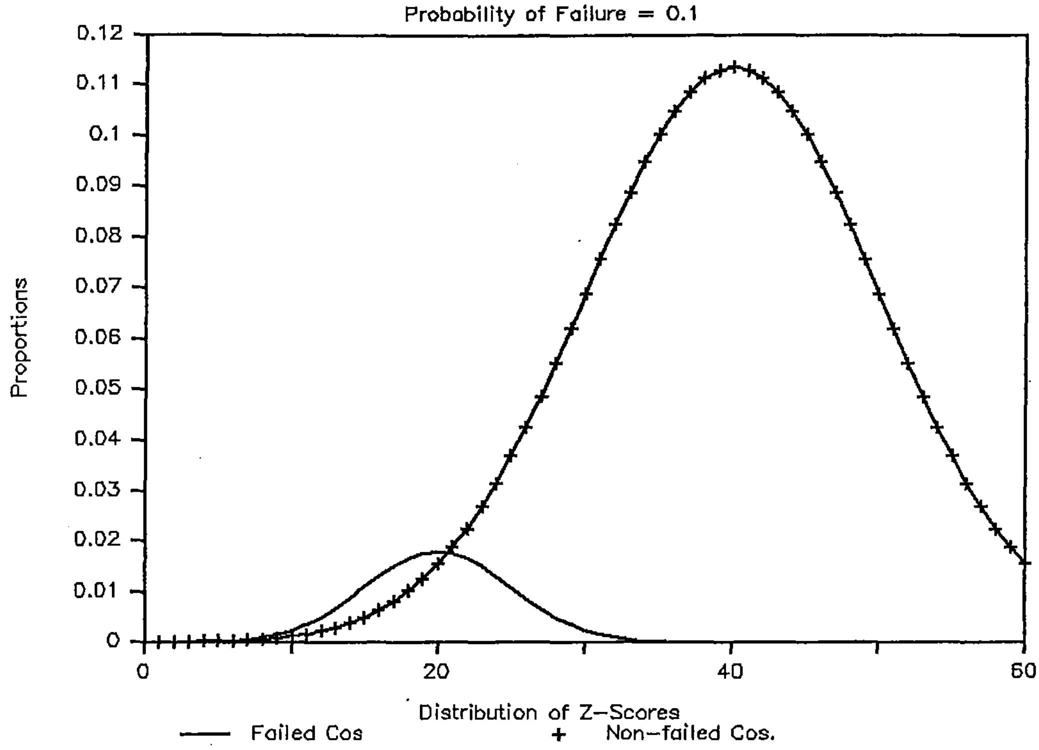
As we move to an *a priori* probability of failure at the 30% level, the model becomes much less satisfactory. Figure 17 shows this quite dramatically. It is a preferable situation to that shown in figure 14.

FIG.17:Covariance Failed <Non–failed Co



Finally figure 18 shows that we have lost most of our discriminatory power in the situation in which the probability of failure is 10%.

FIG.18:Covariance Failed <Non-failed Co



The situation in which we would achieve the kind of unsatisfactory results shown in Figures 15 and 18 is one in which there are many companies with a wide dispersion in their ratios ranging from excellent to very poor. For some reason, other than that which is reflected in the data, a significant proportion do not fail, and others do. In this case we would need to search for an additional variable or variables in order to improve our model's specification. Our model is inadequate as it is. If we extend this analysis by separating the mean vectors even further we would be able to produce a model which does discriminate under these particular conditions. This may take us into the realm of absurdity where the discriminant model is merely telling us the obvious.

CONCLUSION.

Sample derived multiple linear discriminant models of corporate failure may fail to correctly classify the population if *a priori* probabilities are not taken into account. The conclusions to be drawn are quite simple, but important.

1. It is important to identify the population distribution of z-scores for failed and non-failed companies. *A priori* probabilities should be used in this process.

2. The effect of using *a priori* probabilities is to move the cut-off point [if one exists] in the direction of the z-scores representing failed companies.

3. In the case of equal variance - covariance matrices, the extent to which clear discrimination is able to be achieved after *a priori* probabilities are taken into account, is a function of the significance of the difference between the respective mean vectors of ratios. Where mean ratio vectors are sufficiently well separated, the discriminant function will classify with a high degree of accuracy irrespective of the equality or inequality of the variance - covariance matrices.

4. Where the variance - covariance matrix for failed companies is larger than that of the non-failed companies, equal sample size models will exaggerate our ability to discriminate, unless the mean vectors of ratios are well separated. Even then there may be a loss of discriminatory power. In this case there is an increased likelihood of Type I errors, i.e., we will classify a larger percentage of failing companies as non-failures in error.

5. Where the variance - covariance matrix for the non-failed companies is larger than that of the failed companies, equal sample size models may mislead us even more dramatically. In this case there is an increased likelihood of Type II errors, i.e., we will classify a large percentage of non-failed companies as being likely failures in error. The sample data alone will not allow a correct estimate of Types I and II errors to be made

It is clear that care should be taken to observe any inequalities in the variance - covariance matrices in research into linear discriminant models of corporate distress. As already mentioned this will not always lead to clear solutions as some of the elements of the respective matrices may be equal, as others may be larger and others may be smaller. At this stage it would seem that more effort should be spent on identifying ratios that discriminate in a univariate sense [i.e., mean differences should be significant] before we plunge into multivariate techniques. If we cannot find such ratios, we cannot build multivariate linear discriminant models. Both univariate and multivariate tests of the significance between the means and the mean vectors should be applied. The matter of the interaction of *a priori* probabilities and unequal variance - covariance matrices however becomes even more critical as we examine minor differences in mean ratios for one of the groups. This matter is explored in the next chapter.

Chapter Three

THE INTERACTION OF MINOR CHANGES IN MEAN RATIO DIFFERENCES, *A PRIORI* PROBABILITIES AND UNEQUAL VARIANCE - COVARIANCE MATRICES.

INTRODUCTION:

The preceding chapter introduced the argument with respect to the interaction between the *a priori* probabilities and the variance - covariance matrices and the significance of the difference between the vector of mean ratios or other variables. This chapter attempts to add to the rationale for that set of arguments and develop the ideas and implications further. Unequal variance - covariance matrices for the ratios of failed and non-failed companies affect the estimate of the population discriminant function in two respects. Firstly, if the sample sizes for both groups are equal, or approximately equal, as is the case with most published MDA research in corporate distress, and the actual incidence of failure is markedly different in the population, then this will result in a biased estimate of the discriminant coefficients. This is not the case when the variance - covariance matrices are equal however. Secondly, when the discriminant coefficients are applied to the original data of the two groups, the variance, or the standard deviations of the z-scores for both groups will be different. This variance may lead us to misinterpret both the statistics and the individual z-scores themselves. Finally, this chapter examines the extent to which MDA techniques are sensitive to changes in mean ratios and explores the implications that this sensitivity might have for *ex ante* or intertemporal validation.

THE PROBLEM:

From period to period the mean ratios of failed and non-failed companies usually vary, [e.g. Deakin 1977; Lev 1974]. For those developing linear discriminant models of corporate distress this phenomenon should lead to caution, because, as the mean ratios of a single group of failed or non-failed companies change, either through time as industry norms are well known to do, through either sampling error, or measurement error, quite definite changes take place in the estimates of the population parameters involved. These occur because MDA techniques are very much driven off mean differences and off the group means themselves. This part of the research into why models of corporate failure themselves fail, particularly when applied to intertemporal data, investigates the exact nature of the changes in the estimates of the statistical parameters under controlled conditions. A series of questions related to this issue were asked.

EXPERIMENTAL QUESTIONS:

In the event of a mean ratio of one of the groups of failed or non-failed companies changing, for whatever reason, what happens to the estimates of the population parameters in MDA models?

Secondly, what happens if during this process *a priori* probabilities of group membership are introduced?

Thirdly, if during this process the respective variance - covariance matrices diverge, thus breaking the equality assumption, what happens to the estimates of the population parameters in MDA models of corporate distress?

Fourthly, what are the implications of the findings?

Several numerical simulations were modelled and as the results were similar only one numerical example is cited here. This example sufficiently generalises the findings.

THE EXPERIMENTS:

Two sets of simulation experiments were carried out in order to observe the specific effects of variations in a sample mean ratio for a single group, in this case that of non-failed companies. The observed pattern of behaviour would be the same if the a mean ratio of the failed companies was changed. The first set of simulations assumed equal variance - covariance matrices, and the second assumed that the variance - covariance matrix for the non-failed companies was larger than that of the other. The non-failed group of companies was used to simulate the case of the large variance - covariance matrix on the assumption that non-failing companies were more likely to have larger ratio variances. This research did not set out to investigate the empirical validity of the assumption as it is not important to the argument. The question is important to subsequent research however.

During experimentation the sample sizes were adjusted to allow for *a priori* probabilities of group membership which might reflect the population from which the sample was drawn. Because much research in the corporate distress modelling area uses equal sub-sample sizes, the experimentation process commences at this point.

DEFINITIONS:

The vector of sample means were defined as:-

Means of Failed Companies = M_f

Means of Non-failed Companies = M_{nf}

The respective variance - covariance matrices are defined as:-

the variance - covariance matrix for failed companies

$$= \sum{}_f$$

and the variance - covariance matrix for non-failed companies

$$= \sum{}_{nf}$$

The respective sample sizes were defined as:-

n_f and n_{nf} respectively.

The vector of non-standardised discriminant coefficients are defined as:-

$$b = \sum_p^{-1} d$$

where $\sum_p$ is the pooled variance - covariance matrix.

and where d is the vector of mean differences where

$$d = [\, M_f - M_{nf} \,]$$

ESTIMATING THE POPULATION VARIANCE - COVARIANCE MATRIX:

MDA techniques assume that the underlying variance - covariance matrices are identical and that deviations from this are due to sampling errors. Whether or not this is a valid assumption depends upon the particular sub-populations or groups being investigated. From the sample data, the respective variance - covariance matrices are computed, and the assumed population matrix is computed by a pooling or weighted average method. The normal practice of pooling the respective variance - covariance matrices is usually carried out as follows:

$$\sum_p = (n_f + n_{nf} - 2)^{-1} \left[(n_f - 1) \sum_f + (n_{nf} - 1) \sum_{nf}\right]$$

If there are minor differences between $\sum_f$ and $\sum_{nf}$ the matter is of little importance but the problem is that when the sampling proportions are markedly different from those found in the actual population being studied, we have a biased estimate of the pooled variance - covariance matrix because the pooling weights are determined by the respective sub-sample sizes. This in turn produces a bias in our estimates of the z-scores for failed and non-failed companies under certain circumstances and reduces our ability to discriminate satisfactorily.

Experiment I:

Given that $\sum_f = \sum_{nf} = \begin{bmatrix} 4 & 2 \\ 2 & 3 \end{bmatrix}$

The number of ratios = 2, and $n_f = n_{nf} = 50$

$$M_f \quad \begin{bmatrix} 3.00 \\ 1.00 \end{bmatrix}$$

$$M_{nf} \quad \begin{bmatrix} \alpha \\ 2.00 \end{bmatrix}$$

where α is varied from -1.00 to 7.00

Successive simulations involved varying the sampling proportions as follows:

n_f = 40	n_{nf} = 60
n_f = 20	n_{nf} = 80
n_f = 10	n_{nf} = 90
n_f = 05	n_{nf} = 95

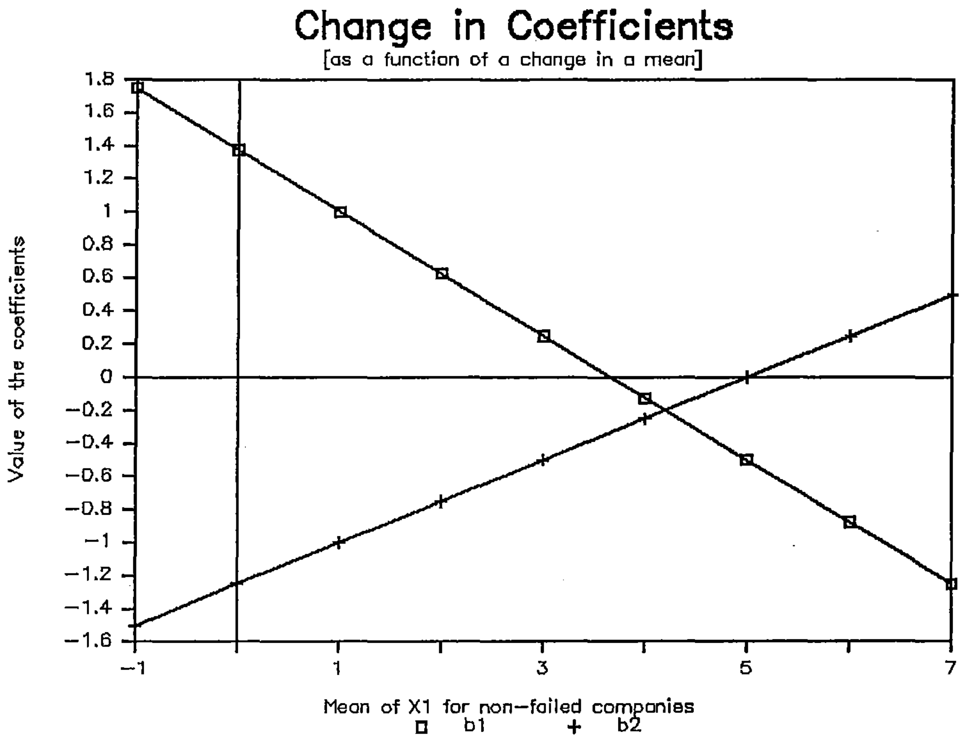
The numerous observations were recorded during the simulations may be classified into two groups. The first set relate to the cases where the respective variance - covariances are equal, and the second to situations where the matrices are unequal. The critical observations are summarised as follows.

I: EQUAL VARIANCE - COVARIANCE MATRICES:

Observation I:1.

As the mean of a single ratio for the non-failed companies group increases, the discriminant coefficients both change in opposite directions to each other because there is a kind of trade-off between the two. Although this particular aspect cannot be generalised, as in some cases both coefficients decrease or increase together, both of these changes are always linear, Figure 3.1.

Figure 3:1



As the mean of X_1, of non-failed companies increases the corresponding discriminant coefficient b_1 reduces in magnitude. Simultaneously the discriminant coefficient b_2 is enhanced. The coefficients relating to these changes appear to be a function of the inverse of the variance - covariance matrices but as the matter appeared to hold little of value to the central purpose of this thesis the idea was not pursued. The particular linear model for both of these linear changes is:-

$$b_1 \quad = \quad 1.375 - 0.375M_{nf1}$$

$$b_2 \quad = \quad -1.250 + 0.250M_{nf1}$$

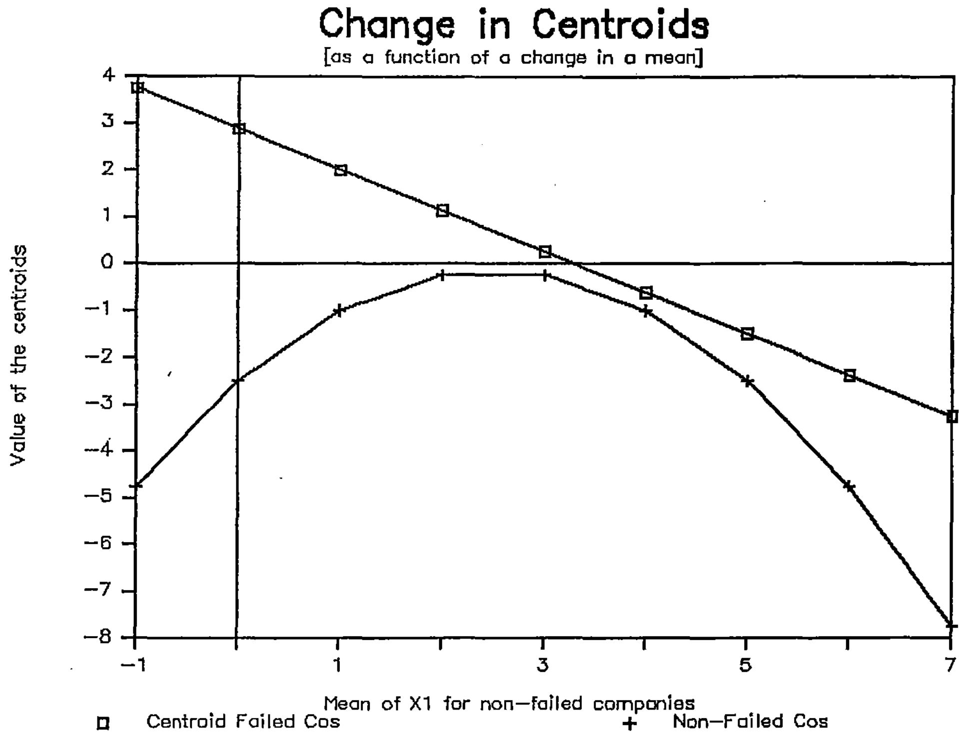
Although more research into this matter could be useful it appears that these linear functions are unique to the specific data set. Again, although more research into this matter could be of interest to mathematical statisticians, it would seem to have little direct interest to those wishing to develop MDA models of corporate distress. It is however important to recognise that there is a trade-off between the discriminant coefficients as the mean ratio of one particular group changes.

The implications of these changing coefficients, particularly in what might be termed, the critical zone, where, in this example, M_1 lies between 3.00 and 5.00, are of particular importance. Not only do small differences in a single mean ratio change both of the discriminant coefficients, but within this range, the respective signs change as well. This means that if, by chance, we select a sample that provided slightly different means from another sample, and to a large extent this would be function of the variance of that particular ratio in the population of companies, we would be inclined to give a markedly different interpretation of the discriminant model. This means that in small sample based research into corporate distress we are constantly in danger of developing sample specific models with relatively large standard errors. Other samples from the same population, with slightly different mean ratios will produce significantly different discriminant models. While at other points, say where the mean of X_1 was 1.0 in the example, the rate of change in the coefficients would be the same as the functions are linear, yet, the interpretation would not be markedly different. A similar change in the mean ratio would not cause the model to have sign changes thus our explanation of the way in which the ratios related to company failure would also not be changed. Clearly more research is needed on this issue but the significance of the trend should be clear.

Observation I:2

As the mean of a single ratio for the non-failed company group increases, the centroids of the non-standardised z-scores also change. The centroid of the failed group changed [reduced in this specific case] in a linear manner while the centroid of the non-failed group changed in a quadratic manner, Figure 3.2.

Figure 3.2



This linear function is:

Failed companies' centroid $= 2.875 - 0.875M_1$

and the quadratic function is

Non-failed companies' centroid $= -2.50 + 1.875M_1 - 0.375M_1^2$

Although these functions would appear to be interrelated in some way they also appear to be unique to the specific data. This matter requires further research in order to identify the nature of this possible interrelationship, however, as it did not appear to be of importance to MDA model builders researching into corporate collapses, the matter was not pursued further. The trends, however, are important.

As the mean ratio changed, the centroids also changed. The centroids of the failed and non-failed companies' z-scores tend towards convergence as the value of the mean ratio of the non-failed group tends towards that of the mean ratio of the failed group of companies. This is logical as when the two mean ratios are equal the particular variable would not contribute to the discriminatory power of the model. In the same vein, as the two means diverge, so to do the centroids and thus the quality of discrimination improves.

Of critical importance here is, that even when the variance - covariance matrices are equal, the MDA technique is very sensitive to changing mean ratio data within particular ranges. Within some ranges the discriminatory power is enhanced or reduced by a significant amount depending upon the magnitude of the difference of the between mean ratios of the two groups. Again this is particularly critical where the mean ratios might have large standard errors. The sampling error is a function of both the sample size and the standard deviation, and there is an additional source of error and movement to be found in changes of accounting policies as well. This might point to the necessity of having a much closer look at the individual mean ratios and their associated standard errors. It is vitally important to ensure that the mean vectors are first-rate estimates of the population mean vectors, and hence random sampling becomes important, because the estimates of the discriminant coefficients and the respective group centroids are completely dependent upon this. It is difficult to over-state this point. More research is needed into the question of MDA modelling of corporate distress in this area because of the sensitivity of the technique to small changes within certain mean ranges. At this juncture I would be inclined to speculate that if the sample estimates of the population means have relatively high standard errors, and this may be a matter of judgement or opinion, then it is probably not worthwhile developing MDA models. It might be possible to enhance the precision of particular models by excluding mean ratios with high standard errors, because as indicator variables, they are too unstable.

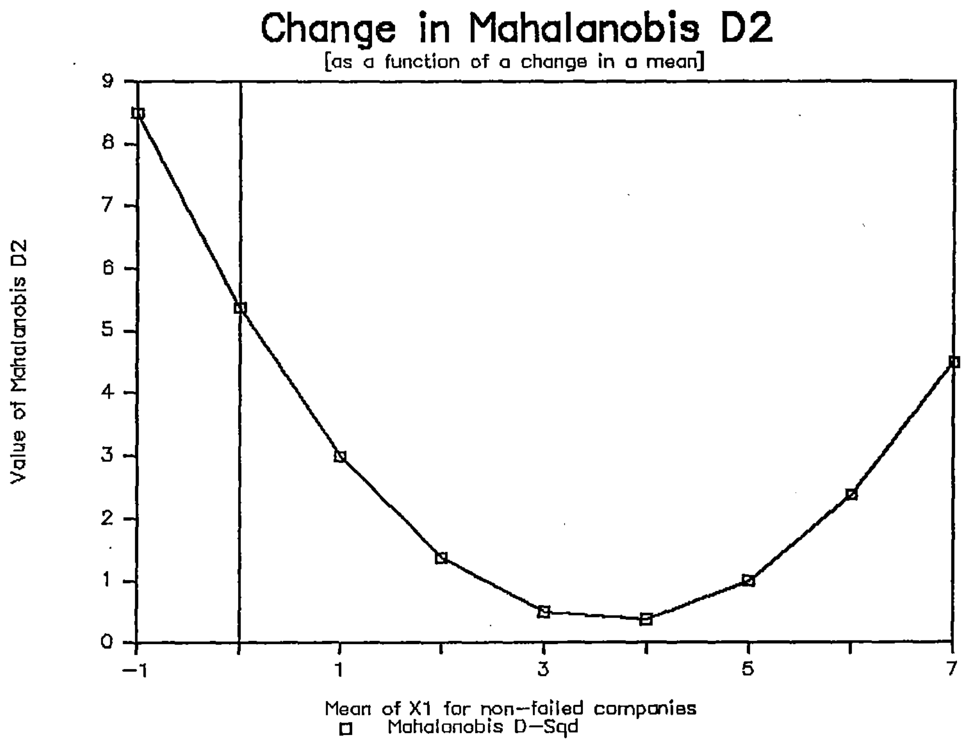
Observation I:3

Now if we subtract the quadratic from the linear equation we obtain a measure of the distance between the two centroids. This is the Mahalanobis δ^2. This provides the function

$$= \quad 5.375 - 2.75M_1 + 0.375M_1^2$$

The usefulness of this function is that we can observe the change in the separation of the centroids and in this case, [figure 3.:3], can observe that between M_1 = 3 and M_1 = 4 there is only a small separation between the group centroids. Discrimination here is very poor.

Figure 3:3

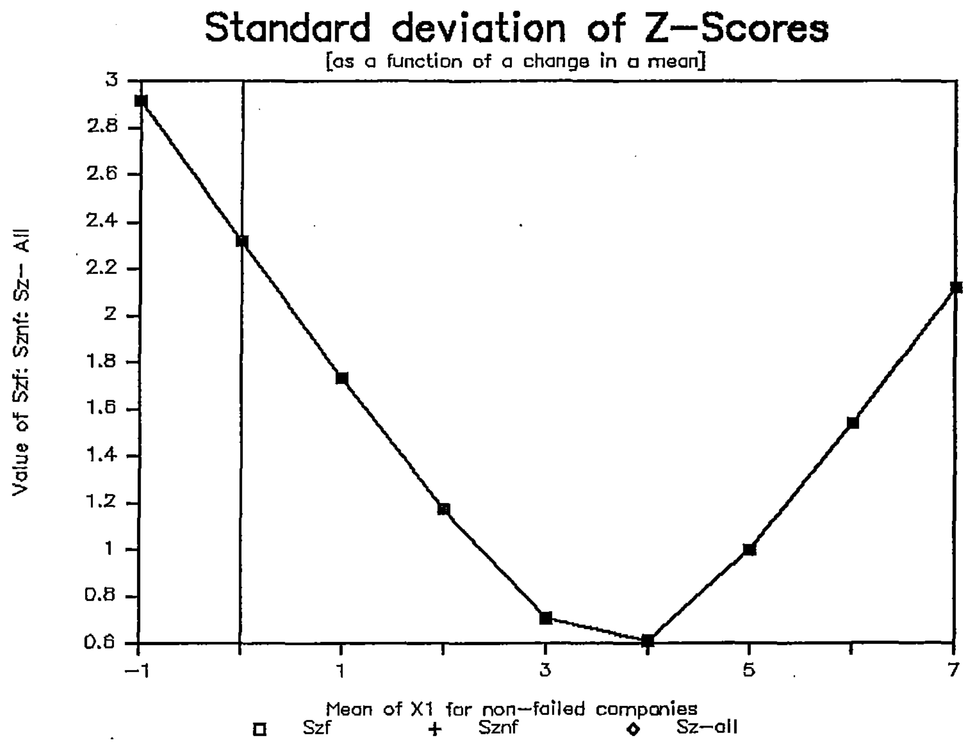


Observation I:4

The standard deviation of the failed, [S_{zf}], the non-failed, [S_{znf}], and the total sample of all companies z-scores,[S_{zts}], are equal and change in a non-linear manner also. That is:

$$S_{zf} \quad = \quad S_{znf} \quad = \quad S_{zts}$$

Figure 3:4



This is to be expected as S_{zts} is the root function of δ^2. Because of the equality of the respective variance - covariance matrices, and as the respective standard deviation of the z-scores are a function of these matrices and the three standard deviations of the z-scores are equal.

$$S_{zf} \quad = \quad b' \textstyle\sum_f b$$

$$S_{znf} \quad = \quad b' \textstyle\sum_{nf} b$$

$$S_{znf} \quad = \quad b' \Sigma_p b$$

Where Σ_f equals Σ_{nf} then it is clear that both of the standard deviations of the z-scores of failed and non-failed companies are equal.

Observation I:5

These first four observations hold true for all *a priori* probabilities. While the variance - covariance matrices are equal, the estimates of the discriminant coefficients, the estimates of the centriods and the associated Mahalanobis δ^2 are exactly the same for all *a priori* probabilities. This has implications for MDA model developers using equal samples of failed and non-failed companies although these proportions are not representative of the population of failed and non-failed companies.

Unfortunately many of these findings do not hold true when the variance - covariance matrices are unequal. This matter is investigated in the following section.

II: UNEQUAL VARIANCE - COVARIANCE MATRICES:

Although statistical theory clearly advocates the use of quadratic discriminant analysis, [Lachenbruch, 1975] when the variance - covariance matrices are unequal, many applied researchers frequently ignore this advice. Several researchers have investigated the use of both linear and quadratic models on the same data and report little difference in their ability to classify distressed and non-distressed companies correctly, [e.g., Altman & Levalle]. Because of the widespread use of linear models with unequal matrices, the affects of changing mean ratios under these circumstances was investigated.

The same scenarios which were examined in the first series of experiments were also examined once more, except that the constraint of the equality of the variance - covariance matrices of the failed and non-failed companies was relaxed. Although the findings are generalisable, the example used to illustrate these findings is one in which the numbers involved are quite specific:

$$\Sigma_f = \tfrac{1}{2} \Sigma_{nf} = \begin{bmatrix} 4 & 2 \\ 2 & 3 \end{bmatrix}$$

The sampling proportions were equal with subsequent trials replicating those of the first set of experimental simulations. In each case, a single mean ratio of the non-failed companies' group was varied, and the results were recorded.

Observation II:1

When the variance - covariance matrices are unequal the discriminant coefficients still change linearly in relation to each other, in a similar manner to the case of equal matrices, as the mean of a single ratio of the non-failed group changes. In this situation, however, there is a different linear transformation for each level of a *priori* probability.

Figure 3:5

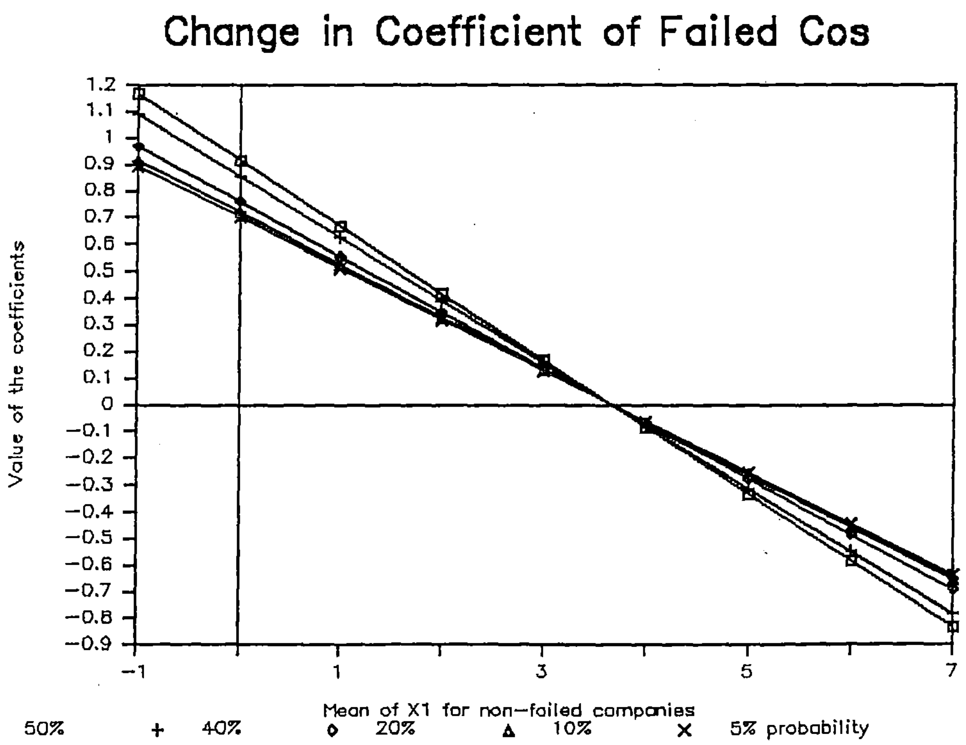
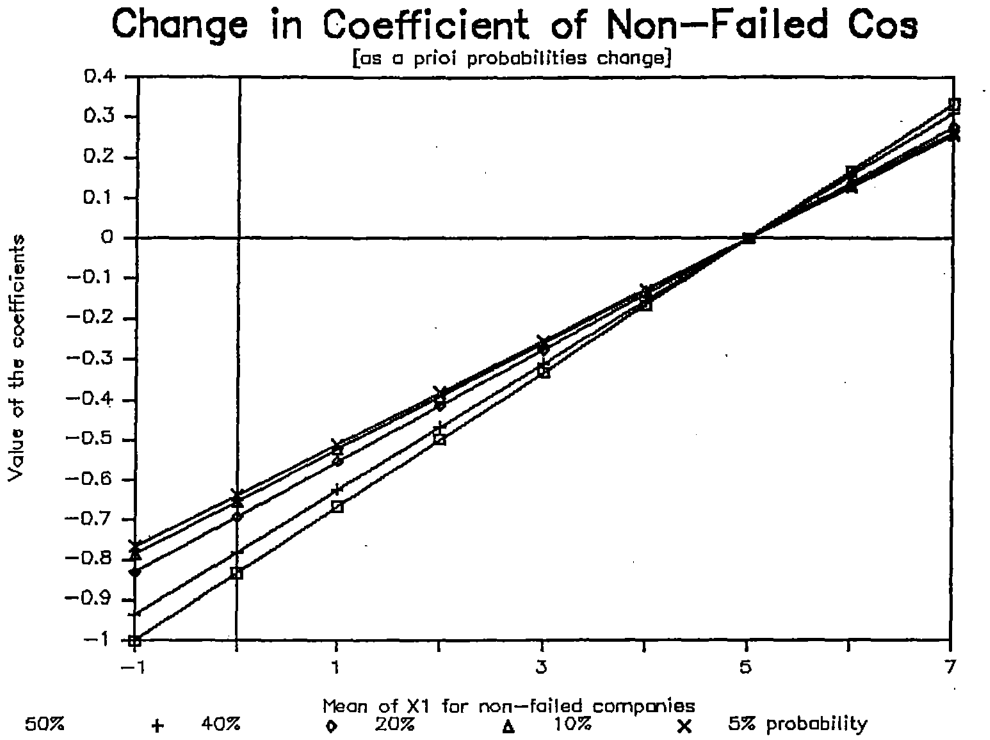


Figure 3:6



Each of these linear transformation functions intersect at:

$b_1 \quad = 0 \quad$ and where $\quad M_1 = 3.675$

and for

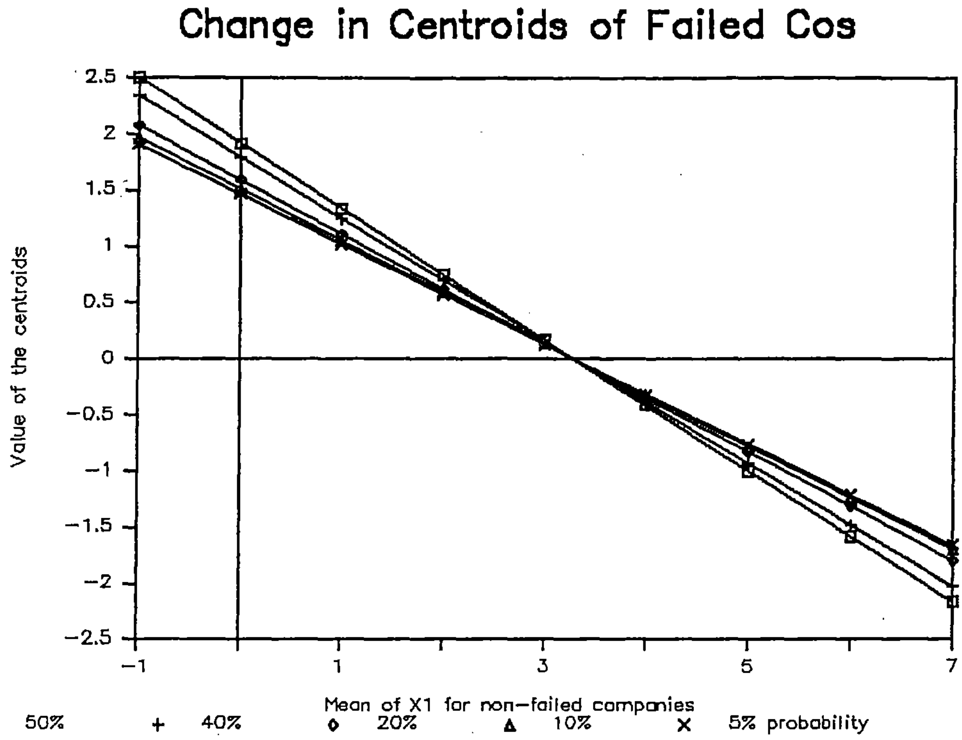
$b_2 \quad = 0 \quad$ and where $\quad M_2 = 5.00$

For each population proportion, or *a priori* probability, there is a different discriminant coefficient. The precise mathematical interrelationships were not investigated as, at this stage, it would appear to be of little import to applied researchers in the field of corporate distress. The implications however are very important for researchers wishing to use multivariate linear discriminant models when the variance - covariance matrices are unequal. Not only is it important to have first rate estimates of the respective mean vectors with low standard errors, but it is important to use a pooling technique which will adjust for the *a priori* probabilities. Even this has to be said with a measure of hesitancy. The complexity of changes that take place, even with minor changes in mean ratios may be significant. In these circumstances it would probably be even more useful to the soundness of any particular piece of research if strenuous efforts were made to find ratios that had equal variances and covariances as well. I am not sure, however, to what extent the pursuit of such ideal data is likely to be fruitful.

Observation II:2

The centroid of the failed company group still changes in a linear manner as it did when there was an equality of variance - covariance matrices, except that there is a different linear function for each level of *a priori* probability.

Figure 3:7

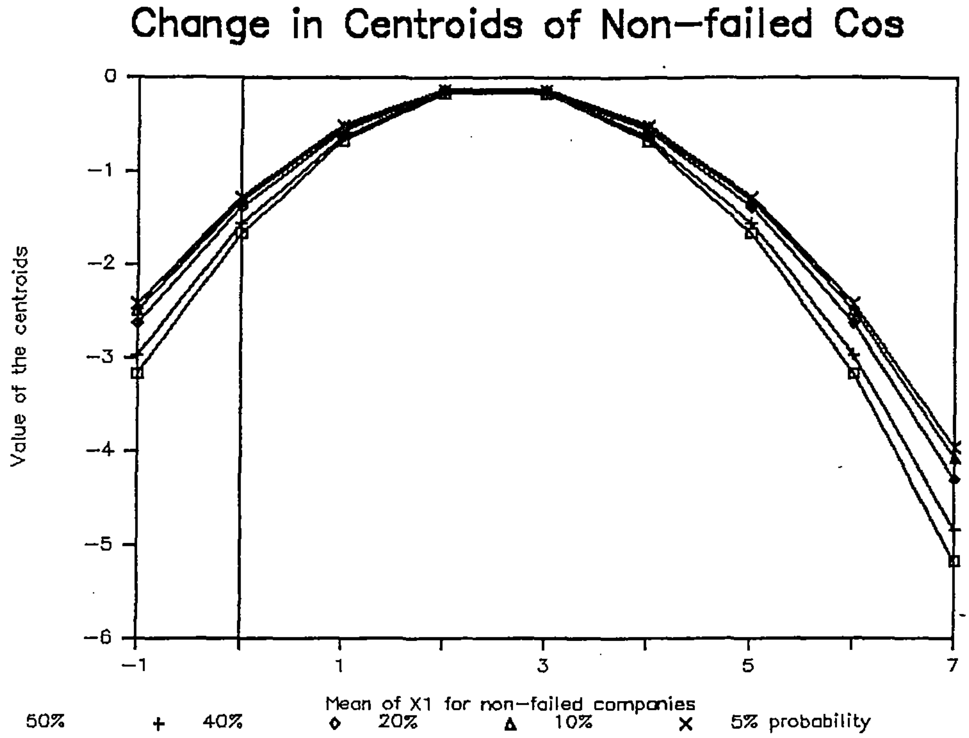


Convergence of each of these functions occurs at:

$$C_f = 0 \text{ when } M_{nf} = 3.275$$

In the same way as found in the first series of experiments, the centroid of the non-failed companies changes quadratically, except that there is a different quadratic function for every level of *a priori* probability.

Figure 3:8



Convergence of each of these functions occurs at:

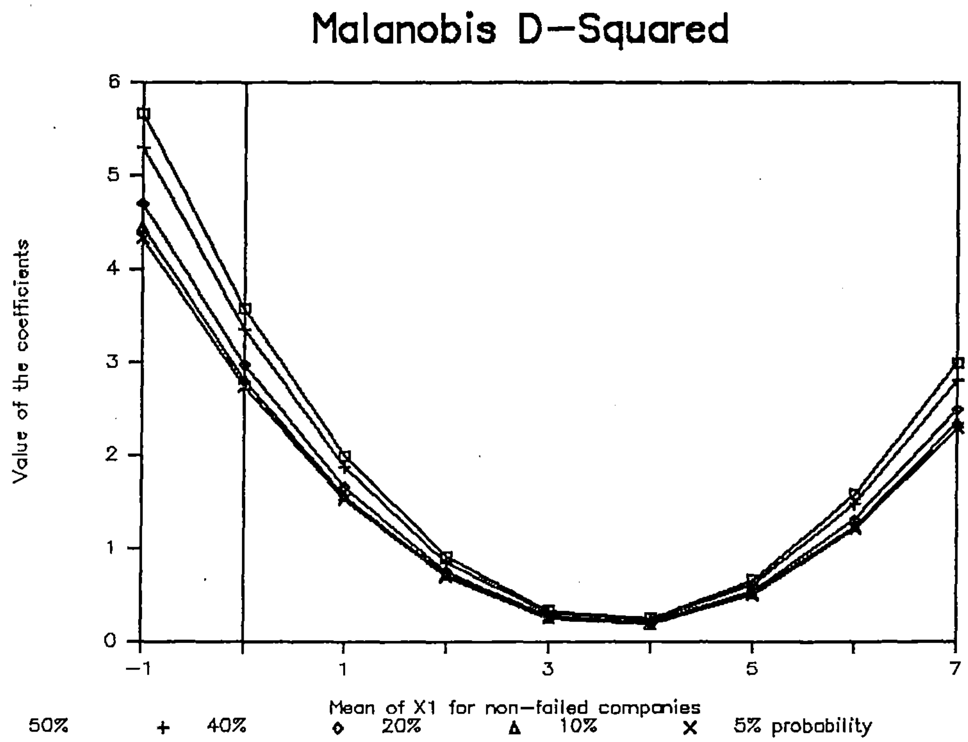
$$C_{nf} = -0.10 \text{ when } M_{nf} = 2.50$$

Observation II:3

The distance measure, the Mahalanobis δ^2, also changes quadratically., but the function itself varies with the changes in the probabilities.

This phenomenon exists because the Mahalanobis δ^2 is the distance between a linearly changing centroid for failed companies and a quadratically changing centroid for non-failed companies, Figure 3:9.

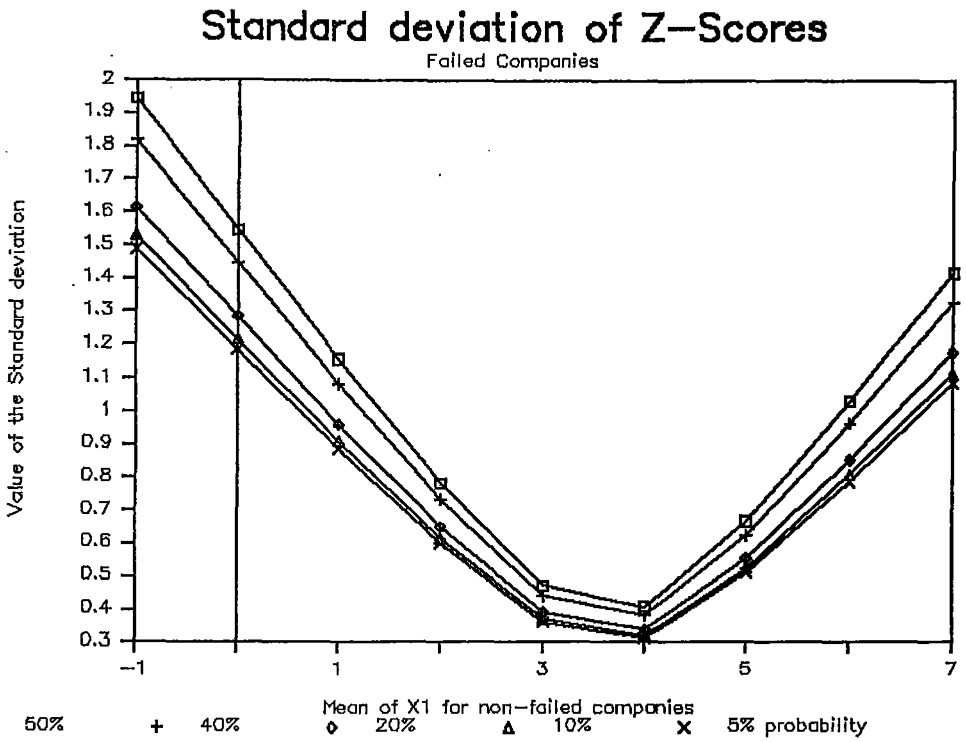
Figure 3:9



Observation II:4

Unlike the case of the equal variance - covariance matrices, the standard deviation of the failed, the non-failed, and the total sample of all-companies group z-scores are not equal. This is because each relates to their respective variance - covariance matrix. The standard deviation of the z-scores for the failed companies is illustrated in figure 3:10.

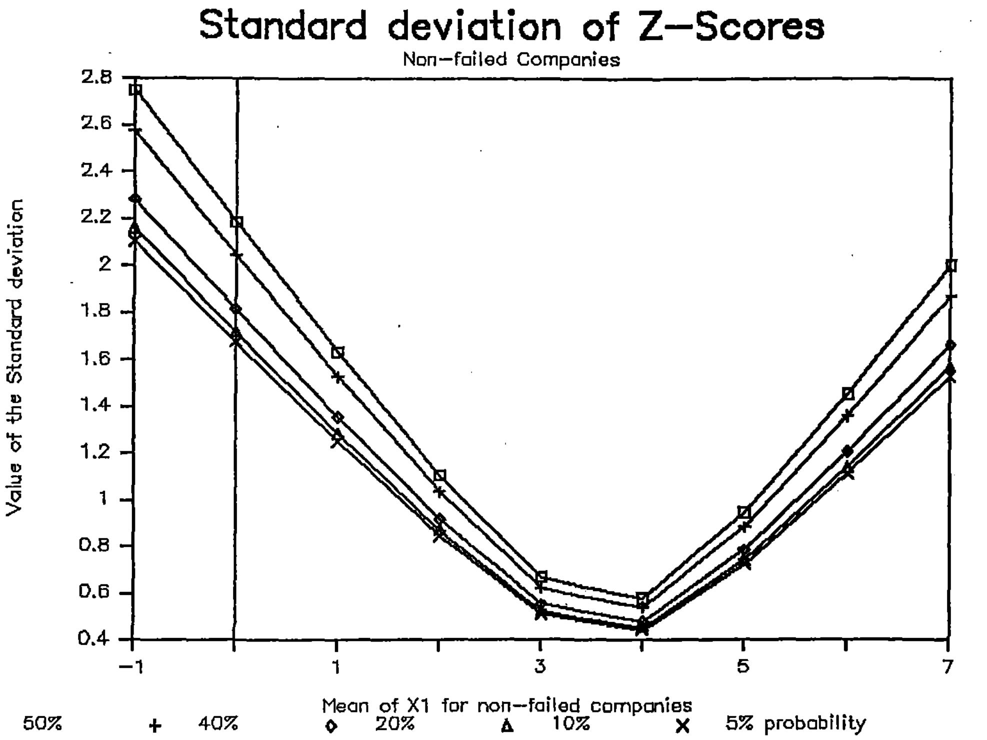
Figure 3:10



That is: $$S_{zf} = b' \Sigma_f b$$

Figure 3:11 shows a similar series of standard deviations for the non-failed company group.

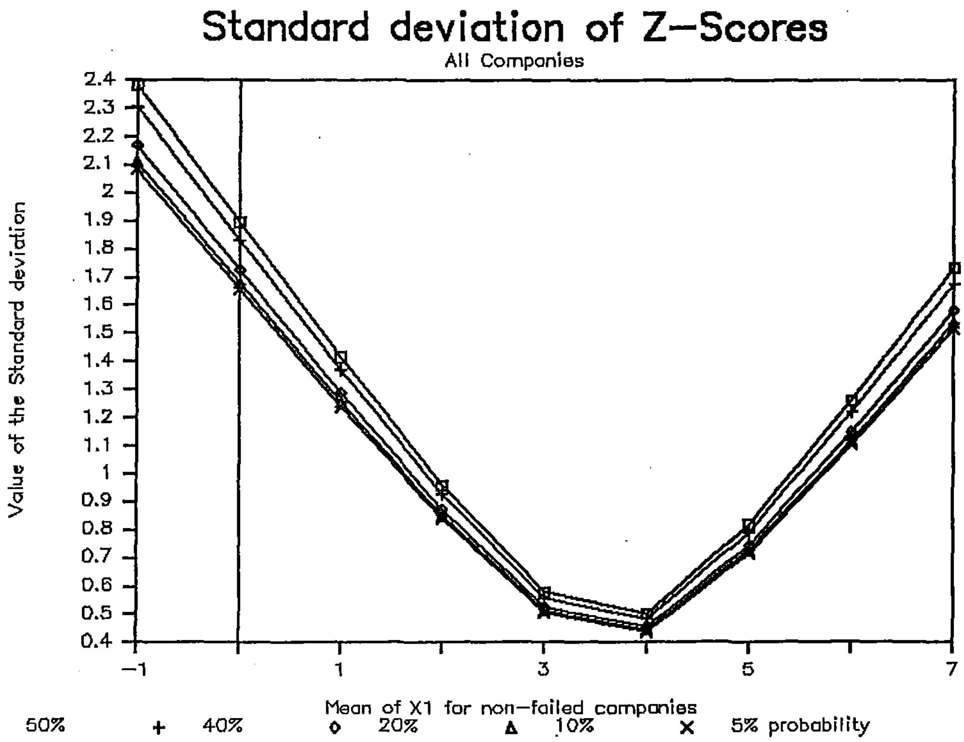
Figure 3:11



That is: $S_{znf} = b' \Sigma_{nf} b$

Figure 3:12 yet again shows the same kinds of trends for the total sample.

Figure 3:12



That is: $S_{zts} = b' \Sigma_{ts} b$

This is a critical issue. As the *a priori* probabilities reduce and in fact approach the proportions commonly found in the general population of companies, the standard deviation of the z-scores of the non-failed companies converges towards that of the total samples. That is this occurs as n_f approaches 5% and as n_{nf} approaches say 95%.

Figure 3:13

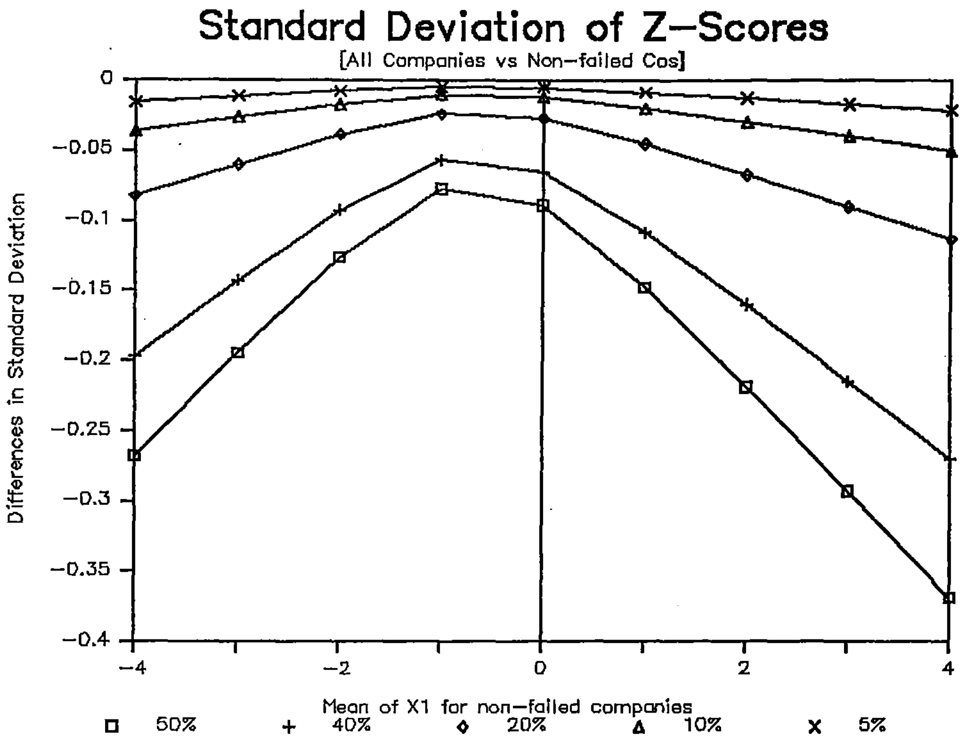
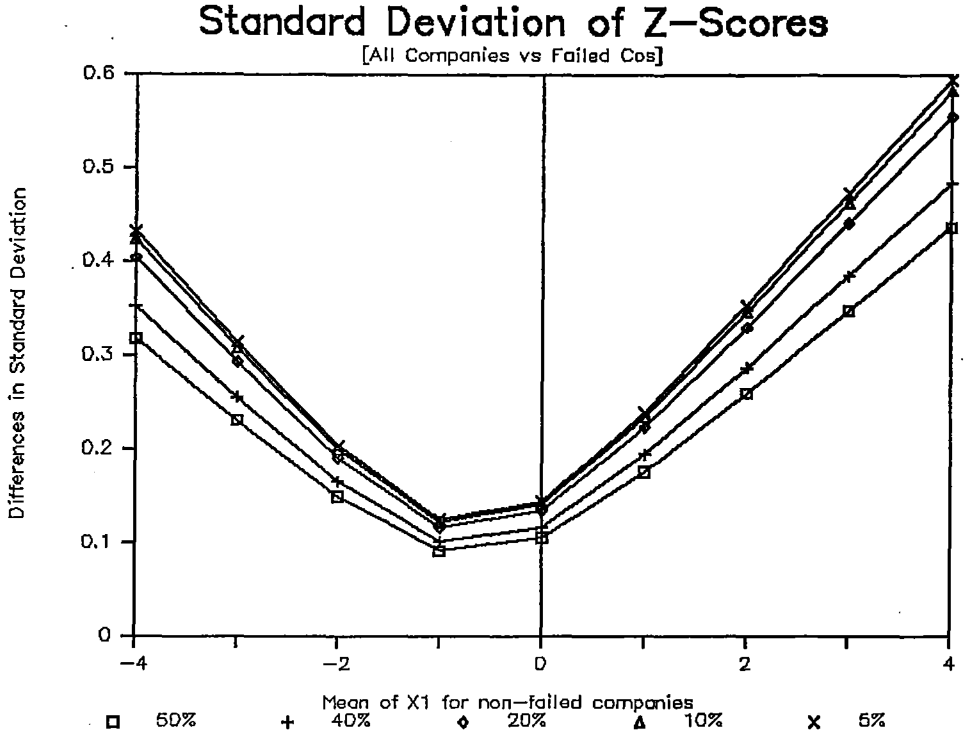


Figure 3:13 shows how the standard deviation of the z-scores for the non-failed companies converges much more closely towards that of the total sample of all companies combined as a *a priori* probabilities of non-failed companies tends towards 100%.

However, although S_{znf} converges towards S_{zts} this is much less the case for the failed group of companies. Figure 3:14 shows how the standard deviation of the z-scores for the failed companies converges, or more correctly fails to converge, towards that of the total sample for all companies combined as *a priori* probabilities of failed companies tends towards a small percentage.

Figure 3:14



SUMMARY OF THE FINDINGS:

The importance of the research into the effect of introducing changes in a single mean ratio for one group of companies in MDA modelling of corporate distress is quite complex. In the case of equal variance - covariance matrices, these changes result in discriminant coefficient and group centroid changes. In the former case there is critical zone outside which there is a tendency to interpret the coefficients consistently as these small changes to the mean ratio are made, and within which not only do the coefficients change but their signs also reverse, thus demanding a completely different explanation. Because one centroid changes linearly and the other quadratically, there is a zone within which small changes in mean ratios are not significant. There is a region outside this zone in which small changes result in markedly different centroids. In this region it is possible to discriminate well.

In the case of equal variance - covariance matrices introducing *a priori* probabilities does not affect the statistical parameters because the pooled matrix is identical to each of those from failed and non-failed companies.

In the case of unequal variance - covariance matrices *a priori* probabilities do affect the statistical parameters quite dramatically. Not only do the changing means provide researchers with similar results as discovered in the first set of experiments, but introducing them also results in changes in a similar but more complex manner. Introducing *a priori* probabilities results in different estimates of the underlying population variance - covariance matrix and thus there is a range of discriminant coefficients, a range of centroids and a change in the dispersion of the z-scores for each level of probability.

In conclusion, it would appear that researchers wishing to use the multivariate linear discriminant model for discriminating between failed and non-failed companies should approach with caution. It is critical that the sample mean vectors should be first-rate estimates of the ratios for the particular industry or population. Without first-rate estimates, the respective statistical parameters, and particularly in the case of unequal variance - covariance matrices, the resulting linear discriminant model will not be a satisfactory estimate of the population MDA model.

IMPLICATIONS FOR FORECASTING FAILURE:

The problems associated with developing MDA models with groups of companies' ratio data whose respective variance - covariance matrices are unequal should be quite clear enough by now. In short, if the matrices are not equal then it is important to use a pooling method that reflects the incidence of company failure in the population. This final section discusses the implications for forecasting company collapses at least one period ahead.

Even in the case of equal variance - covariance matrices, if an MDA model is derived from sample data that perfectly reflects the underlying population mean ratio vectors so that the actual sampling error is zero for all means ratios, applying the model in a future time period, say $t+1$, has one major problem. The extent of this problem may be so devastating as to completely damn virtually any application of our derived model outside the period from which we have drawn our sample.

I can envisage two types of problems. The first is that the same relationships between particular ratios and the likelihood of company failure might not exist in time period *t+1*. Our lack of a complete theory in this respect makes this difficult to assess.

Secondly, if the time period during time period *t+1* is one in which the mean ratio vectors are significantly different, even though the relationship between company failure and its ratios is the same as during the sample time period from which we derived the particular model, it will fail to forecast company collapses correctly. As we have already acknowledged, there is widespread evidence to support the contention that mean ratios are almost invariably different from one time period to the next, [Lev,1974].

Such an error in application would be exemplified in the following situation, even when there are only two ratios and equal variance - covariances. If we begin by deriving a model from sample data from a prior period.

Given that $\Sigma_f = \Sigma_{nf} = \begin{bmatrix} 4 & 2 \\ 2 & 3 \end{bmatrix}$

The number of ratios = 2, and $n_f = n_{nf} = 50$

$$M_f \quad \begin{bmatrix} 5.00 \\ 2.00 \end{bmatrix}$$

$$M_{nf} \quad \begin{bmatrix} 4.00 \\ 3.00 \end{bmatrix}$$

The discriminant coefficients would be $b_1 = -0.625$ and $b_2 = +0.750$ with centroids of $C_f = -0.250$ and $C_{nf} = -1.625$.

Now even if the mean vectors of ratios for the period *t+1* both change in the same direction and by the same magnitude, such that the mean vector of differences remains the same, and even if the variance - covariance matrices remain the same as in the derived sample, such that the discriminant function coefficients would remain the same, we will have a considerable amount of forecast error.

Given $M_f \quad \begin{bmatrix} 7.00 \\ 3.00 \end{bmatrix}$

$$M_{nf} \quad \begin{bmatrix} 6.00 \\ 4.00 \end{bmatrix}$$

We would have the same vector of mean differences as in the previous period, but the means of the z-scores of the failed and non-failed companies would be markedly different. They would now be C_f = -0.750 and C_{nf} = -2.125. As the sample means of the ratios for the period *t+1* increase, the mean z-scores of the failed companies tends towards the centroid of the non-failed companies thus rendering discrimination and therefore *ex ante* validation impossible. This would mean that we would most probably forecast all of the companies as being non-failed.

We might be tempted to argue that if we know this degree of change then we could adjust for the change in mean vectors and effectively rescale the ratios. The problem is that we do not have this information in advance. We do not know which companies are likely to fail and which are not, for that is the purpose of the modelling. Therefore we cannot have access to the information about the respective group means.

Chambers [1973] adds to our understanding of the problem when discussing the extensive array of accounting practices that makes forecasting difficult. The revaluation of assets, or a change in the method of depreciation, to name but two areas of diversity, not just between companies, but within them, exacerbates our difficulties. *"The freedom of each company to choose its own rules and to vary the rules it uses at its discretion may abort any attempt to benefit from financial features of companies,"* [p.90]. He continues to argue that *"a little exercise on the combination of these and other permissible rules and methods will show that there are over one million sets of rules, each* *of which could be said by managers and auditors to give a true and fair view of a company's state of affairs and its results. The odds against the financial statements of any two or more firms being comparable are enormous,"* [p.91]. Chambers provides a wealth of examples. This makes the search for ratios which reflect a *"proper"* view of the health of a company extremely difficult, and, furthermore, provides a compounded difficulty when it comes to establishing stable mean ratios for failed and non-failed companies from which to develop MDA models of corporate distress. Chambers make the point that companies can and do choose methods of measurement that frequently disguise the underlying facts.

Finally, the situation just described was very controlled. The mean ratios are unlikely to move in tandem and the variance - covariance matrices may not remain as they were in the period from which we derived our model. Accordingly, when this occurs it is difficult to predict likely outcomes, there is a need to look closely at the specific data. Under these conditions several coefficients might otherwise be different.

CONCLUSION:

There is clearly an interaction between changes in mean ratios, a *priori* probabilities and unequal variance - covariance matrices. Where the latter are equal, the dispersion of the z-scores of both groups are equal, but where equality of matrices does not exist I would suggest that a *priori* probabilities should be reflected in the sampling proportions so that the pooled dispersion matrix might *best* approximate *reality*.

It is also clear that the MDA model, when applied to distinguishing between failed and non-failed companies is very sensitive to mean ratio changes irrespective of the extent of the equality of the variance - covariance matrices. The possibility of mean ratios changing from one period to another is likely to be a major reason for intertemporal validation failure. Lev [1974] shows this to be the case. The extent to which mean ratios change in period $t+1$, for example, will be reflected in the extent to which the model will fail to forecast correctly, even if there is the same relative relationship [i.e., the mean differences remain the same] between particular ratios and company success or failure.

Chapter Four

THE INTERACTION OF NON-MULTIVARIATE NORMALLY DISTRIBUTED DATA AND UNEQUAL VARIANCE - COVARIANCE MATRICES.

INTRODUCTION

Although research into the effect of unequal variance - covariance matrices using multivariate normally distributed data has been carried out by Gilbert, [1969], it appears that little work has been published about univariately normally distributed data or non-normal data that might typically be found in company ratios, [Lachenbruch, 1975]. Researchers attempting to build linear discriminant models of company failure, are frequently confronted with an additional problem to those already discussed in this research.The data may not be multivariately normally distributed. In fact, I am unaware of any reported research in this field that claims that the data involved is distributed in this manner. The procedures for evaluating the extent to which the data is multivariately normally distributed are outlined by Watson, [1990], but the best that researchers can often do with the sample ratio data that is frequently scarce, or at best difficult to obtain, is to transform the ratios into unvariately normally distributed variables and carry out quite simple checks to see whether or not the transformation has been successful. While McLean and Firth [1987], for example, were able to show that although New Zealand company ratios were not even univariately normally distributed, it was possible to transform the data into such univariate distributions. Freka and Hopwood [1983] have also shown this to be the case with the ratios of U.S. manufacturing firms for the period 1950 - 1979. Whether this procedure is theoretically justifiable is a question which lies outside the central concern of this discussion. Of vital practical importance to empirical researchers using the MDA technique is the question of the extent to which the use of Fisher's linear discriminant analysis is invalidated by the use of non-multivariate normally distributed data. Several researchers, [e.g. Eisenbeis, Jones], have been critical of reported research that does not have multivariately normally distributed data. As part of a series of explanations of why models of corporate distress fail, and at times why they seem to work despite breaching the assumptions, this chapter discusses research into to which univariately normally distributed data, combined with both equal and unequal variance - covariance matrices will allow satisfactory discrimination to take place. Although it is important to identify the circumstances under which this is possible, the findings do provide some hope for the practice of using non-multivariately normally distributed data together with unequal variance - covariance matrices in corporate distress prediction.

THE OBJECTIVE:

If data is multivariately normally distributed, then the individual variables will be univariately normally distributed. The converse does not hold. Data may be univariately normally distributed, but not multivariately so, [Tabachnic & Fidell, 1983]. It is probably too much to hope for that ratio data commonly used in corporate distress modelling will be unvariately normally distributed, let alone expect it to be multivariately normally distributed. Gilbert [1969], Marks and Dunn, [1972] and subsequently Lachenbruch, Sneering & Revo, [1973] estimated the effect that unequal variance - covariance matrices have on the ability of Fisher's discriminant function to correctly classify observed phenomena. In these cases, however, they usually assumed the multivariate normal distribution. With known multivariate distributions it is possible to derive solutions and associated probabilities of misclassification. As they are generally not reported in MDA research into corporate distress we can only suspect that the multivariate distributions of ratio data used in many pieces of research throughout the world are many and varied. What is needed is an investigation into the extent to which it might be possible to use the MDA technique satisfactorily with the kind of data commonly used in distress prediction modeling. This research attempts to proceed some way down this pathway. The objective of this part of the study is to investigate the extent to which simulated ratio data, which is univariately but probably not multivariately normally distributed, can be correctly classified as the variance - covariance matrices of failed and non-failed companies diverge. If we can show that this relationship is fairly predictable, we can identify in advance the kinds of situations in which models of corporate failure are likely to fail to classify adequately. If we know this, we can take steps to search for satisfactory solutions.

THE METHOD OF INVESTIGATION

The method of investigation involved establishing a series of random variables which were univariately normally distributed for each of two groups. These represented the ratios of failing and non-failing companies. The method by which this was achieved is described later in this section. As with the work of Gibson, and Marks & Dean, the second set of variables representing those of non-failed companies were a linear transformation of the first. This enabled both the mean vectors and the variance - covariance matrices to be altered under controlled conditions. The mean vectors of ratios were gradually separated, in standardized increments of 0.25, by a simple linear transformation, and, at each stage the effect on the ability of the MDA technique to classify companies correctly was recorded. As a part of this process, the variance - covariance matrices were also separated by a linear transformation of the variances and covariances in such a way that the ratio of the respective sizes took the following values 1:1, 2:1, 3:1 and 4:1. Each stage was carried out in a controlled, or step-wise, manner so that the results could be plotted as other factors were held constant.

The simulated ratios for failed companies were generated by way of a BASIC language random number generator RND which produces numbers according to a rectangular distribution. These numbers were then converted to a univariate normal distribution by way of Hamming's [1962] method. The specific programming code for this routine is listed in lines 210 - 340 in the simulation programme, [Appendix 4:4]. After the generation of the random numbers the data was analysed by way of a multiple linear discriminant programme. The programme used was a modified version of James' [1985] discriminant programme for two groups, [Appendix 4:4]. The modification was largely confined to the routines involved with data input and output so that the simulation process could run for long periods of time without requiring attention.

The sample size in each simulation was 500 non-failed and 500 failed companies. Two, three, four, ten and twenty ratio models were evaluated. Except for the twenty variable model each of the other four models involved ten separate trials using different sets of randomly selected data. The complete set of percentages of correct classifications for the four ratio case is reported in appendix 4:1. The mean percentages for each of the ten simulation runs for each of the two, three, four and ten ratio cases are reported in appendix 4:2. The standard deviations of the simulated mean ratios are reported in appendix 4:3. Because of the excessive amount of computer time required to run the simulation programme, the twenty ratio model was only run once. The data are reported in appendix 4:5.

AN ANALYSIS OF THE RESULTS OF THE SIMULATION

The results are reported in two stages. The first involves a detailed analysis of the investigation into ten sets of simulated sample ratio data, each with four variables. The second part of the report provides much less detail, but is much more comprehensive in its scope in order to provide a sounder basis for generalising. It discusses the results of the simulations of the two, three, ten and twenty ratio studies.

Firstly, the four ratio model was chosen quite arbitrarily for a start. The variables, X_1 to X_4, were chosen and the normally distributed data set with 500 cases in each group was obtained as described earlier. The data was then standardised so as to have means of zero and initially a standard deviation of one. This allowed for simpler linear transformations without influencing the outcomes of the experiments. The variance-covariance matrices of failed and non-failed companies were equal. A typical one is reported in table 4:1.

Table 4:1 - $\Sigma_f = \Sigma_{nf}$ - Four ratio case.

	X_1	X_2	X_3	X_4
X_1	1.00	0.04	-0.04	-0.12
X_2	0.04	1.00	-0.07	-0.06
X_3	-0.04	-0.07	1.00	0.07
X_4	-0.12	-0.06	0.07	1.00

As the data was standardised, the variance - covariance matrix was also the correlation matrix.

A priori probabilities throughout the experiment were set at the 0.5 level. It should be of no great surprise to find that the function classified correctly only 50% of the time when the mean ratios of both failed and non-failed companies were identical.

The experimental process involved successively increasing the standardised means of the ratio data for the non-failed companies in increments of 0.25. This was carried out in order to record the percentage of correct classifications as the distance between the two vectors of mean ratios increased. The original results are reported in appendix 4:1. Table 4:2 summarises the mean percentages of correct classifications obtained from the 10 simulation runs. Appendix 4:3 reports the standard deviations about the overall means reported in table 4:2. As the dispersion of simulated results is very low indeed with a maximum of 1.81% for all simulations, the overall means of the 10 simulations can be regarded as being highly representative of the recorded data.

Table 4:2 - $\Sigma_f = \Sigma_{nf}$ - Four ratio case.

Standardised Mean Vector Differences.									
0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00	3.00
Percentage of Correct Classifications.									
50%	62%	72%	81%	88%	92%	94%	97%	99%	100%

Clearly, this is the kind of situation for which Fisher designed MDA. Where the variance - covariance matrices are equal, the ability of the technique to discriminate between the two groups is enhanced as the difference between the mean vectors increases, despite the lack of conformity to the multivariate normality assumption. Ignoring the question of *a priori* probabilities and the question of the costs of misclassification, we clearly have fairly good discrimination where the mean vectors are separated by one standardised unit, i.e., an 88% correct classification. It seems that the assumption of multivariate normality appears only to be critical where the mean vectors are markedly similar. Clearly as the differences between the means of the ratios of the two groups increases, discriminatory power increases dramatically. Any failure of MDA models to discriminate is a function, not of the technique itself, but of the characteristics of the data. If the mean ratio data is significantly different for each group, and if that data is stable over time, and, as already discussed in chapter three, this is likely to be a problem, then there does not seem to be any reason for MDA models not to classify correctly. It may be the failure of the many researchers in the field of corporate distress to understand this point that has caused them to discard MDA models unnecessarily. I strongly contend that the search for a better technique is misguidedly based upon the wrong assumption, that is, the assumption that there might be something less than satisfactory about the MDA technique. If there are minor differences between the ratios of the two groups, and the respective sample variances are relatively large, then our ability to discriminate is limited, if at all possible. If not, the Fisher model will discriminate.

As we relax the equality constraint with respect to the variance - covariance matrices to the point where the variance - covariance matrix of the second group is twice the size of the first, the normal pooling procedure is to used to estimate the population variance - covariance matrix. Like other computing programmes, James' programme also utilizes this approach. The resulting matrix is shown in table 4:3.

Table 4:3 - $\Sigma_f = 0.50\ \Sigma_{nf}$ - Four ratio case.

	X_1	X_2	X_3	X_4
X_1	1.5	0.06	-0.06	-0.18
X_2	0.06	1.5	-0.11	-0.09
X_3	-0.06	-0.11	1.5	0.11
X_4	-0.18	-0.09	0.11	1.5

As we know under these conditions we should use the quadratic model, [Lachenbruch, James], yet from a careful observation of table 4:4 we can see that the MDA model classifies very well after the standardised mean difference becomes 1.25. Even at a standardised mean difference of 1.0 we have only lost 4% on our previous classification rate.

Table 4:4 - $\Sigma_f = 0.50\ \Sigma_{nf}$ - Four ratio case.

Standardised Mean Vector Differences.									
0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00	3.00
Percentage of Correct Classifications.									
50%	61%	70%	78%	84%	90%	91%	93%	95%	100%

The resulting variance - covariance matrix of the second group is three times the size of the first group we have the following results:

Table 4:5 - $\Sigma_f = 0.33\ \Sigma_{nf}$ - Four ratio case.

	X_1	X_2	X_3	X_4
X_1	2.0	0.09	-0.09	-0.24
X_2	0.09	2.0	-0.14	-0.11
X_3	-0.09	-0.14	2.0	0.15
X_4	-0.24	-0.11	0.15	2.0

The percentage of correct classifications is shown in table 4:6

Table 4:6 - $\Sigma_f = 0.33\ \Sigma_{nf}$. - Four ratio case

Standardised Mean Vector Differences.									
0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00	3.00
Percentage of Correct Classifications.									
50%	61%	68%	76%	81%	88%	90%	92%	93%	98%

Again at the mean difference level of 1.25, for example, it is still possible to achieve the same level of correct classification as achieved with the first model at a mean difference of 1.0. Finally we can examine the case where the variance - covariance matrix of the second group is four time the size of the first table 4:7.

Table 4:7 - $\Sigma_f = 0.25\ \Sigma_{nf}$ - Four ratio case.

	X_1	X_2	X_3	X_4
X_1	2.5	0.11	-0.11	-0.30
X_2	0.11	2.5	-0.18	-0.14
X_3	-0.11	-0.18	2.5	0.19
X_4	-0.30	-0.14	0.19	2.5

The percentages of correct classification is show in table 4:8.

Table 4:8 - $\Sigma_f = 0.25\ \Sigma_{nf}$ - Four ratio case.

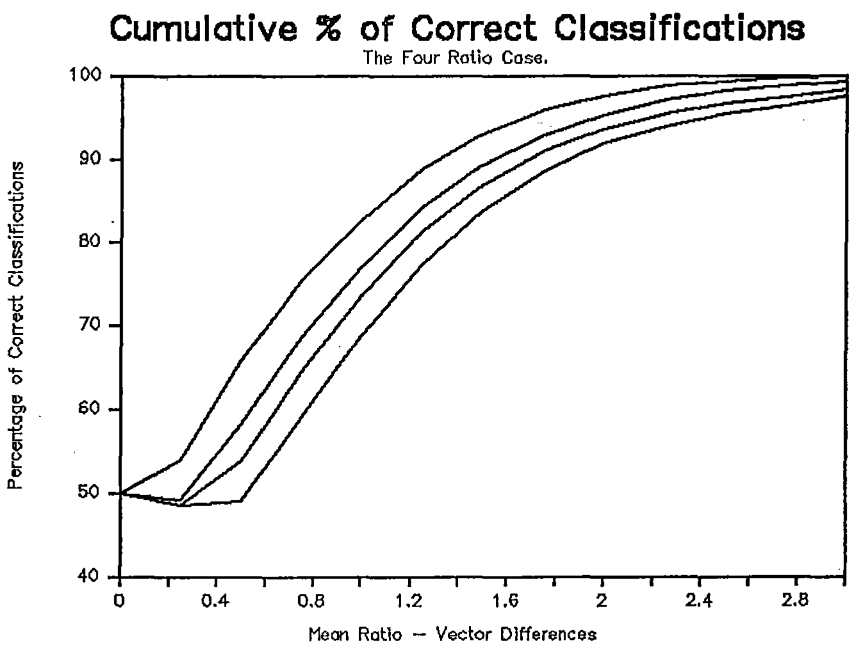
Standardised Mean Vector Differences.									
0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00	3.00
Percentage of Correct Classifications.									
50%	59%	67%	75%	82%	86%	89%	91%	92%	96%

The four ratio case shows the extent to which classification accuracy is possible as the variance - covariance matrices diverge from each other. Appendix 4:2 reports the same kind of results from the simulations involving two, three, ten and 20 ratios. A graphical summary of the finding should allow an understanding of the interaction between the variables in the process.

A GRAPHICAL PRESENTATION:

If the results are collated into a graphical summary [figure 4:1] of the effects of differences between the vectors of mean ratios in relation to the unequal variance - covariance matrices, we can observe the trend for the first sample set of four variables.

Figure 4:1



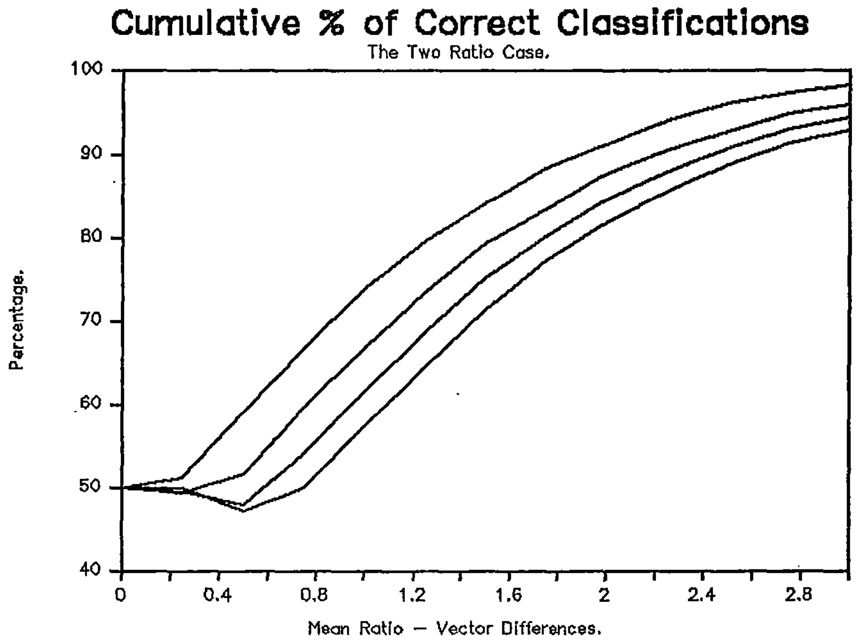
The graph line on the left represents the mean percentage classification for the equal variance - covariance case, where $\Sigma_f = \Sigma_{nf}$, while the curve on the right the case where the variance - covariance of non-failed companies is four times that of failed companies, i.e., where $\Sigma_f = 0.25\ \Sigma_{nf}$,. The two intermediate graph lines represent the intermediate relationships accordingly.

It appears that despite the supposed need for a quadratic discriminant approach, researchers are able to obtain satisfactory results from the MDA technique despite the fact that the variance - covariance matrices are clearly unequal. The extent to which this is possible is a function of the differences between the mean vectors as well as the magnitude of the difference between the variance - covariance matrices. As with the question of the relevance of *a priori* probabilities, the critical factor influencing the situation is that of the differences between the means of the ratios for the failing and non-failing companies respectively. If we have relatively large differences then *a priori* probabilities are far less important and similarly if we have relatively large differences between the ratio means, we have clear discrimination arising from a linear discriminant model, despite the inequality of the variance - covariance matrices, and despite the fact that the data is only univariately normally distributed. Although the evidence seems to justify this conclusion in the case of a four variate model a more generalised base is required to show that the results are not unique to this situation.

A MORE GENERALISED APPROACH:

In order to confirm the conclusions already drawn four sets of similar simulations were carried out. Two, three, ten and twenty ratio models were evaluated. The trends were of a remarkably similar kind to those produced by the four ratio model. Figure 4:2 reports the findings of the two ratio model.

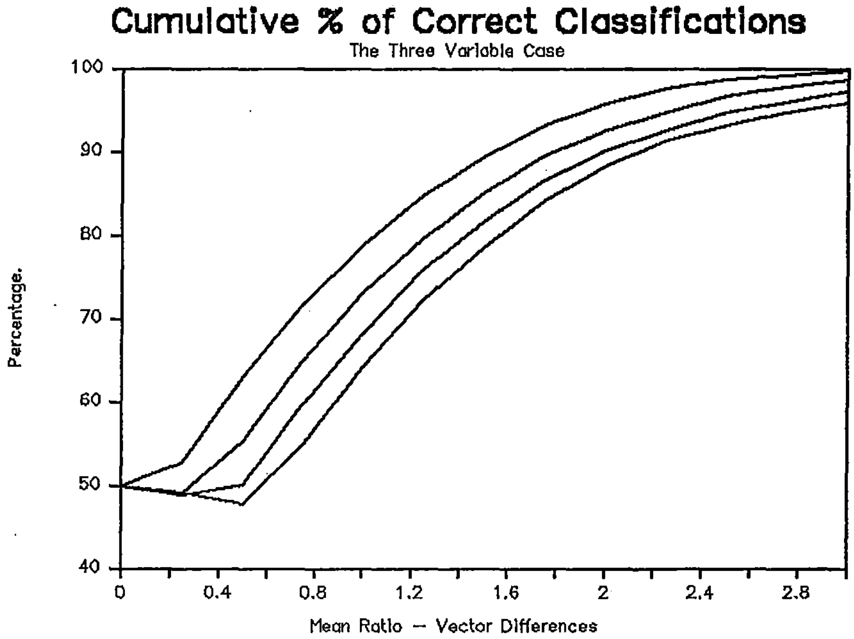
Figure 4:2



In each case the upper curve reflects the cumulative percentage of correct classifications where the variance - covariance matrices are equal. The next curve down reflects the percentage of correct classifications where $\Sigma_f = 0.5\ \Sigma_{nf}$. The remaining two curves are the results of the simulations under conditions in which $\Sigma_f = 0.33\ \Sigma_{nf}$ and $\Sigma_f = 0.25\ \Sigma_{nf}$ respectively. The two ratio model provides similar results to the four ratio model. Next three ratios were evaluated under the same conditions.

Figure 4:3 reports the findings of the three ratio simulations.

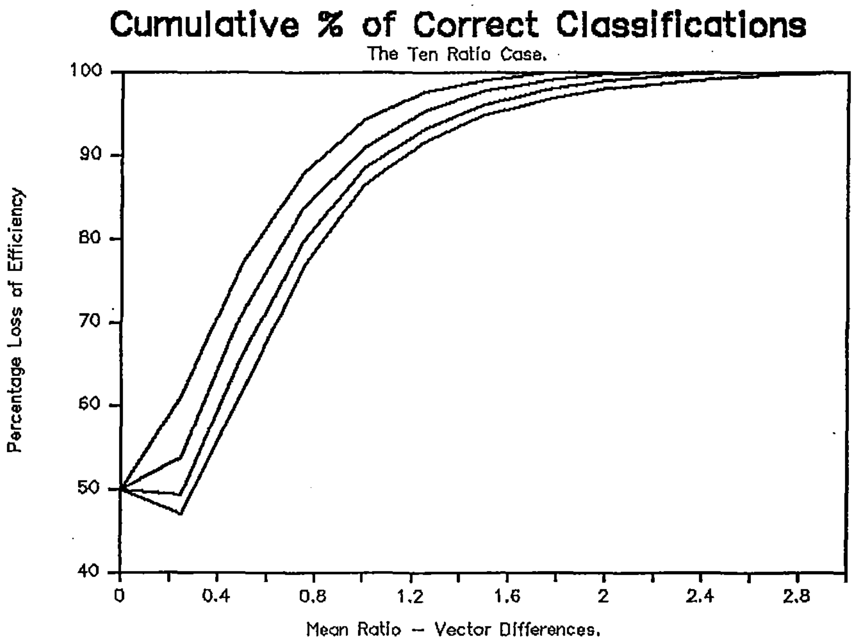
Figure 4:3



Again a similar pattern of relationships between the mean ratio vector differences and the respective variance - covariance matrices was obtained. As a result of the similarity of results with models involving two, three and four ratios it was decided to investigate a ten ratio model under the same conditions.

Figure 4:4 reports the findings of the 10 ratio simulations.

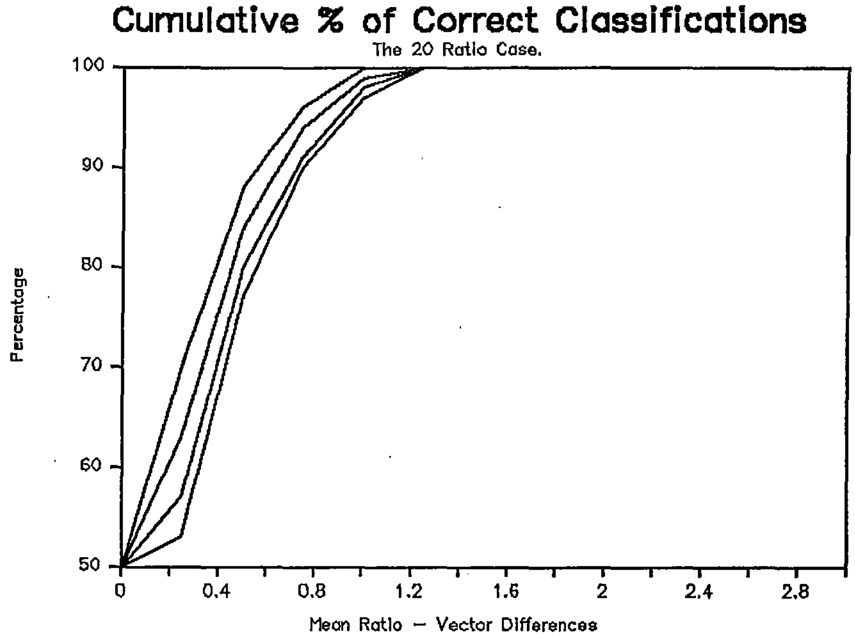
Figure 4:4



Apart from the tendency of the curves to move upwards and to the left as the number of ratios was increased the results obtained from the simulations were very much the same. A final run with 20 ratios was made in order to draw final conclusions.

Because of the 20 ratio simulations required a large amount of computer time only a single simulation was undertaken. This was no doubt because of the slowness of the BASIC language programme. The slowness of the language did not of course affect the findings. The results which trend in the same manner are reported in figure 4:5.

Figure 4:5



Each of the models produces the same pattern of results. There is an initial inability to classify above the probability levels of chance, around the range of mean differences between 0.0 and 0.80, until the mean vectors are separated sufficiently. After this high percentages of correct classification are rapidly achieved. In each case the ability of the MDA model to classify correctly is enhanced with smaller differences between the variance - covariance matrices of failed and non-failed companies. This is reflected in the fact that in each case the differences in the ability to classify correctly become smaller as the mean vectors separate. This is what we would expect in the case of multivariately normally distributed data. It is reassuring to find that this is also the case with ratio data that is only univariately distributed. The fact that we do not know the distribution behind the data however means that with small samples we are unlikely to have any idea of the distribution of the z-scores and hence are unable to be able to speak meaningfully about probabilities of group membership if there is anything like a *zone of ignorance*.

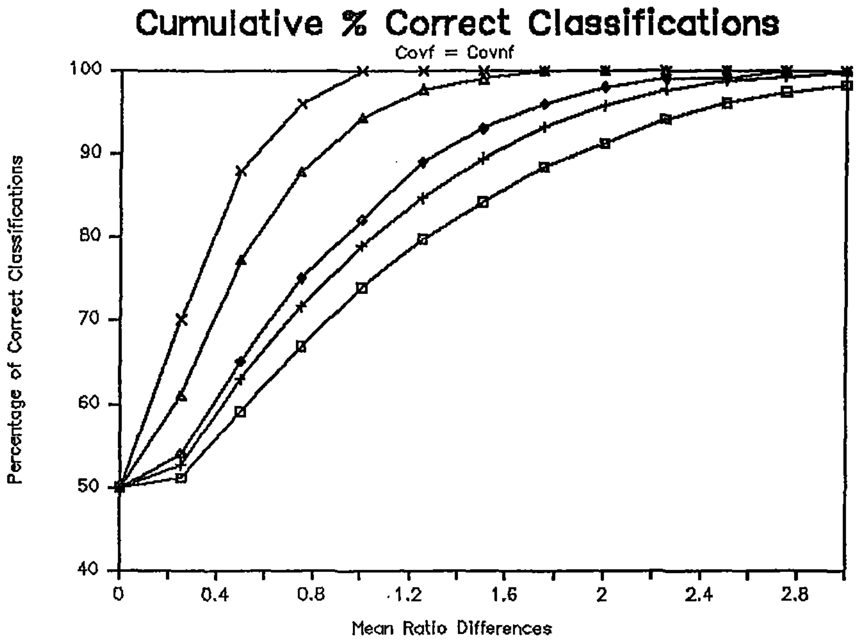
In this section of this report the results of varying both the variance - covariance matrices, the mean differences and the number of ratios have been reported while holding the number of ratios constant in each graph. The following section reports the results while holding the variance - covariance matrices constant in order to obtain a clear view of the affects of varying the number of ratios in the modelling process.

<u>COMPARING THE SITUATIONS DIFFERENTLY:</u>

If we now take the same information and compare the extent to which the number of ratios in the models are able to provide adequate discrimination while the variance - covariance matrices are separated we gain yet further insights.

The simulations in which both matrices were equal are summarised by the graph in figure 4:6

Figure 4:6 $\Sigma_f = \Sigma_{nf}$



Where:

x = the 20 ratio model

Δ = the 10 ratio model

$\diamond$ = the 4 ratio model

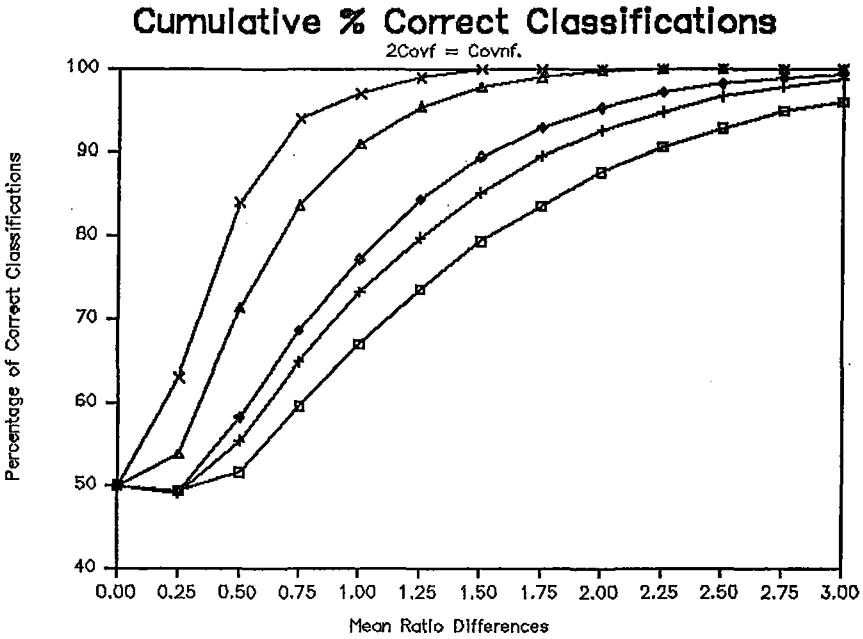
+ = the 3 ratio model

$\square$ = the 2 ratio model

Although the data is only univariately normally distributed all of the models rapidly classify the simulated ratio data correctly.

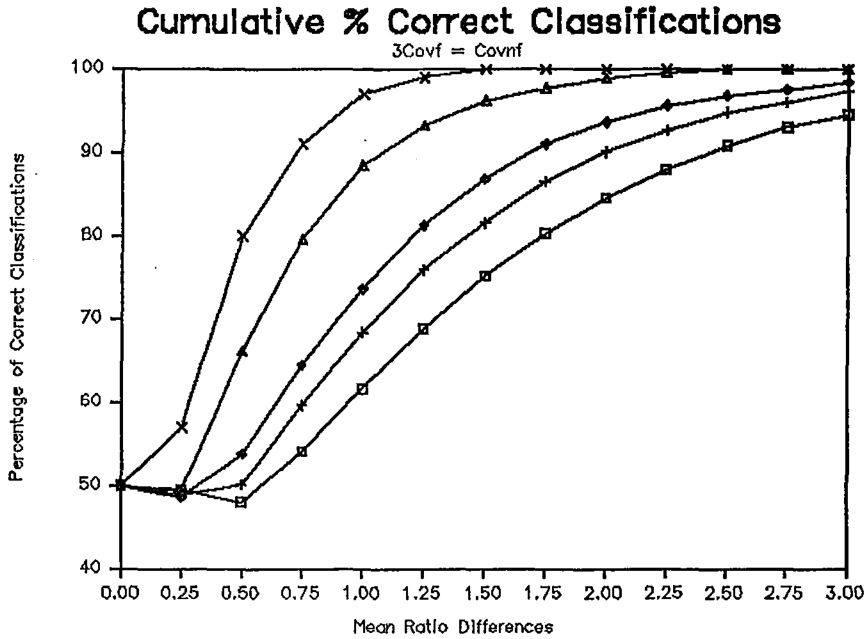
The simulations in which the variance - covariance matrix of failed companies is twice the size of that of the non-failed companies is portrayed in figure 4:7.

Figure 4:7 $\Sigma_f = 0.50\ \Sigma_{nf}$



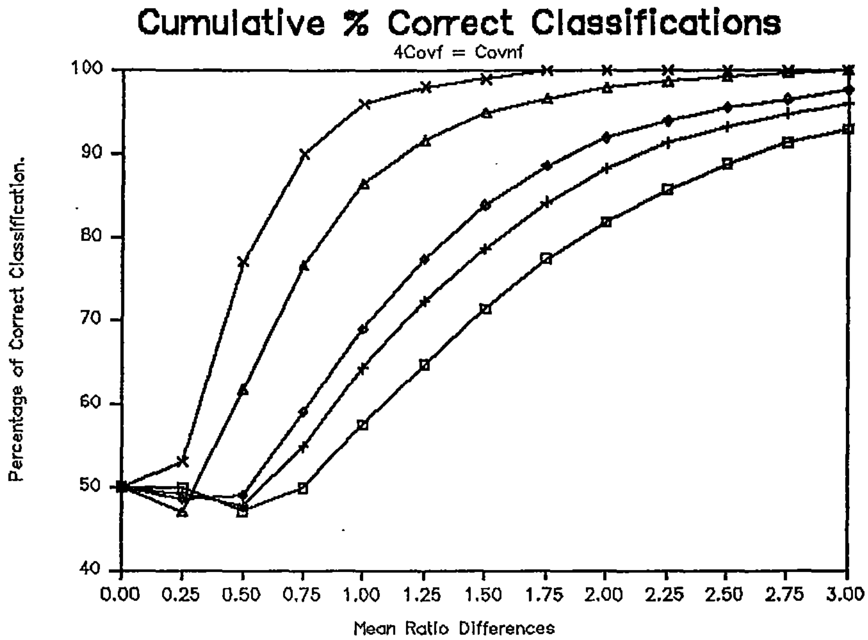
Clearly, in the case where the variance - covariance matrix of the non-failed companies is twice the size of that of the failed companies, the ability of the MDA model to discriminate between the two groups is not only possible but it improves progressively with an increase in the number of ratios.

Figure 4:8 $\Sigma_f = 0.33\ \Sigma_{nf}$



The same pattern is reflected in the data where the variance - covariance matrix of the non-failed companies is three times as large as the other. Finally, what might be termed the extreme case, where the variance - covariance matrix is four times as large as that of failed companies, the same pattern is revealed, [figure 4:9].

Figure 4:9 $\Sigma_f = 0.25\ \Sigma_{nf}$



These three graphs lead us to similar conclusions. There are a few important points to recognise in these findings.

Firstly, the ability of the MDA model to classify correctly deteriorates progressively as the variance - covariance matrices diverge. The graph in figure 4:9 has a similar pattern to that of figure 4:6 but the curve has shifted to the right.

Secondly, in each of the three graphs, the graph lines are rather jumbled where the mean differences are relatively small. This reflects the fact that, as the variance - covariance matrices diverge from each other, larger mean ratio vector differences between failed and non-failed companies are required before linear discriminant models begin to classify correctly.

Thirdly, once the differences between mean ratios vectors are clearly different the models all classify well, irrespective of the fact that the variance - covariance matrices are not equal, and irrespective of the fact that the data is only univariately normally distributed. It seems that linear discriminant models classify well in the extremes. Critical comments by various authors [e.g. Eisenbeis] regarding the apparent misuse of MDA techniques when the assumptions of multivariate normality and the equality of the variance - covariance matrices may be valid where the mean differences are not relatively large. In these kinds of circumstances the MDA models will fail to classify satisfactorily, but where the mean ratio vectors are well separated MDA models appear to classify well despite the breach of the formal assumptions.

Fourthly, the more ratios in the model, the more successful the MDA model is at correctly classifying the data even when the variance - covariance matrices are markedly different. From a certain critical level of mean difference between the ratio vectors the curves rise steeply as the models rapidly increase their ability to discriminate. This critical level which changes according to the extent of the inequality of the variance - covariance matrices can be observed on each of the graphs. In all cases the larger the number of ratios the more steeply the percentage correct classification curve rises from this critical point. This does not provide a basis for arguing that step-wise procedures should not be used in order to reduce the data in any way, particularly as the simulated data was only very weakly intercorrelated. Step-wise procedures only reduce the dimensions of discriminant models where multicollinearity is present. Judging from the culling out of ratios from various published models there would seem to be a fairly high degree of multicollinearity in the variables used. This is logical because the ratios are drawn from balance sheets and income statements and frequently have the same denominators. As multicollinearity increases amongst the variables the less successful we will be in correctly classifying as additional variables are included.

LIMITATIONS OF THE STUDY:

Although I am confident of the conclusions of this part of the study, the study itself has a number of limitations, some of which point to the need for further research, others of which may not be too important.

Firstly, it was been assumed that the univariately normally distributed data was not also multivariately distributed. The simulations had been completed when it occurred to me that it would have been a little more reassuring to have tested for multivariate normality. As obtaining such a multivariate normal distribution, even one, by chance however, in so many simulated trials [41 trials in total] would be extremely unlikely, I think that the results are still valid.

Secondly, like Gilbert's research methodology, the two sets of simulated data were related in that the ratios of the non-failed companies were a linear function of the ratios of the failed companies data. This was done in order that the variance - covariance matrices could be exactly equal to each other for the first set of trials. Although I cannot perceive anyway in which this might invalidate the results, nor Gilbert's, it is at least possible that this may be the case.

Thirdly, although this part of the research has focused on the univariate normal distribution, other distributions of ratio data need to be simulated. By analogy the results should be the same or similar.

Fourthly, although *a priori* probabilities were not used to develop the model, this would affect the cut-off points in the various models, but perhaps more critically, as was found in chapter three, as the variance - covariance matrices diverged the discriminant function itself would change. This aspect needs further research.

Fifthly, in this study all of the mean ratios of the non-failed companies were increased simultaneously. This was done in order to identify the trend in the ability to classify correctly. This is important but in reality I strongly suspect that this is unlikely to happen for there will probably tend to be a confusing set of movements.

Although we cannot be totally sure, the limitations of the research do not appear to be too damning. It appears that valid conclusions may be tentatively drawn.

CONCLUSIONS:

It is clear from these experiments that despite the non-multivariate normality of the distribution of the simulated ratio data the critical factor in whether or not discriminant models of corporate failure are likely to be able to discriminate satisfactorily is the differences between the mean vectors of ratios. If we wish to produce useful linear discriminant models of corporate failure we should focus on identifying ratios which are significantly different. Before plunging into discriminant analysis we need to test the significance of the difference between the mean vectors of ratios for the failing and non-failing groups of companies. Ratios whose means are not significantly different will not discriminate well. Sets of ratios whose means are significantly different will discriminate well, even if the data is only univariately and not multivariately distributed, and even if the variance - covariance matrices are not equal. It appears that the greater the inequalities in the mean vectors of ratios, although there is still room for more research in this field, irrespective of the underlying distribution of the ratios, the less important the question of the equality of the variance - covariance matrices becomes. Finding ratios, however, that are markedly different for failed and non-failed companies is not necessarily an easy task.

The criticism that MDA models of corporate distress are generally in breach of the assumptions underlying the technique is valid enough. It is difficult to find fault with this. However, from this research, it appears to be a much less critical issue than has been argued in the literature.

Chapter Five

SAMPLING FAILURES

INTRODUCTION:

In general, poor sampling methods have been employed in past research into discriminant models of corporate distress. They are poor in several important respects and each contributes to the ultimate failure of the MDA models. The most critical is the failure to use random samples. Except where population data is used, unless sample companies are selected by random methods, then *ex post,* let alone *ex ante* evaluation, is highly likely to be a waste of time. The selected samples are most unlikely to be representative of the population because of biases in selection. Other sampling issues have also been widely discussed and will be summarized in the brief literature review. This chapter introduces two more design failures in the sampling methods employed. They appear to be important. Firstly, I will argue, that although seemingly scientific, the common practice of dividing a sample into a *hold-out* sample and a sample from which the MDA model is derived, the method is at best only pseudo-scientific and at worst completely unscientific. It is pseudo-scientific in that hypotheses are not first established and then tested by replication. It is also pseudo-scientific in that the *hold-out* method involves such a vast number of possible combinations that models that seemingly replicate in this fashion are really only encountered by chance. This matter is discussed at length. Secondly, the common practice of using a *hold-out* sample is inefficient because the standard errors of the ratio means in any sample of companies involved is increased unnecessarily in the process. The sort of data needed for this kind of research is scarce enough without our compounding the problem by using it inefficiently.

LITERATURE REVIEW:

The sampling methods have been extensively criticized by several writers. Joy and Tollefson [1975] made the point that proper random sampling should be used where we expect the results to be representative of any population larger than the sample. "*The sampling frame should be conceptually identical to the populations towards which the research question is directed*", [p:725]. They point out, for example, that if it is intended to discriminate between good and bad loan applicants the "*samples should be from populations of good and bad loan applicants, not from populations of good and bad loan acceptances where applicants that were denied credit are excluded,*" [p:725]. The more we deviate from random sampling, the more bias is allowed to develop and the more the derived model is likely to be sample specific. Although much of the research carried out in this field has been with non-randomly selected samples, this aspect has been adequately examined in the literature, [Zimjewski 1984]. Although sometimes population data is used, the discussion in this chapter assumes that proper random samples have been selected with a view to deriving an MDA model of a larger population of which the particular sample is representative.

Zimjewski [1984] "*examines conceptually and empirically two estimation biases which can result when financial distress models are estimated on nonrandom samples. The first bias results from "oversampling" distressed firms......The second results from using a "complete data" sample selection criterion*", [p:59]. His argument about the oversampling bias is discussed because of the almost overwhelming tendency for researchers to use the population of failed companies in contrast to a sample of non-failed firms is interesting. Although he was able to show that considerable biases existed in the probit and logit models, he found that only the constant was biased in the case of MDA models.

Zimjewski also points out that although the employment of the "*complete data*" criterion in sample selection might seem to possess certain niceties, this leads to biases in the selection process. Companies for which only incomplete data is available are excluded from the analysis. In reviewing Zimjewski's paper, Dietrich [1984] reinforces the notion with a complementary thought. "*Bankrupt firms have significantly longer delays in releasing financial reports than nonbankrupt firms,*" [Dietrich 1984; p:86]. This exacerbates the sampling errors involved.

Jones [1987] also discusses sample selection in data collection. "*An unfortunate outcome of the focus on bankruptcy is that sample sizes may be quite small,*" [p:133]. For example, Altman [1968], Deakin [1972], Elam [1975], Altman et.al., [1977], Van Frederikslust [1978], Norton and Smith [1979], Dambolena and Khoury [1980], Emery and Cogger [1982], and Hamer [1983], rely on small samples of bankrupt firms numbering only 33, 32, 48, 53, 20, 30, 46, 52, and 44 respectively. He points out that there is a sampling bias in that large bankrupt firms are always included, while small firms are not even included in the US Compustat database. "*The importance of this bias is undeniable since small firms are especially prone to bankruptcy,*" [p:133]. He also points out that new firms are usually excluded from the sample because researchers typically want five or more years of data "*in order to test the capability of the model to forecast failure five or less years away,*" [p:133]. The fact that new firms are usually excluded when they are especially prone to financial collapse, introduces yet another source of bias into the sampling processes employed.

Finally, Jones argues that although the use of *matched pairs* seems to be sound in some respects, this also presents problems. "*An unfortunate drawback of this approach is that the controlled variables [used in the matching process] may be valuable predictors,*" [p:134]. The matching approach may be excluding important distinguishing qualities.

The list of sampling errors and biases introduced into MDA research into corporate distress is quite substantial. This chapter discusses two further sampling issues so far not discussed in the literature. The first is the inadequacies of replicating with a *hold-out* sample. It is at best pseudo-scientific. The second is the inefficiency of using an *hold-out* sample in the process. It increases the standard errors involved unnecessarily.

THE INADEQUACIES OF REPLICATING WITH A HOLD-OUT SAMPLE:

The objective of scientific enquiry is to establish the facts about relationships in the real world. An important part of the establishing of this *truth* is to be able to demonstrate these relationships repeatedly; i.e., by replicating research findings. This process usually involves a serious attempt at specifying an hypothesis, which seems to provide some explanation for observable relationships in the real world. Having established an hypothesis by evidence, the model is then usually re-evaluated using a fresh sample of data. The hypothesis is then accepted, rejected or modified for further research. This process is rarely followed in the development of particular models of corporate distress.

The most commonly used method by which MDA models of corporate distress have been developed throughout the world is one in which a sample of failed and non-failed companies have been selected. In order to derive a model it is common practice to divide the sample into two smaller sub-samples, the latter being the *hold-out* sample and the former being the data from which the model is derived. The ratios, recorded against each company, are frequently analysed by way of a step-wise linear discriminant programme applied to the first sub-sample. This step-wise procedure normally culls out superfluous variables and reduces the multicollinearity problem amongst the ratios. The *derived* models are then tested against the *hold-out* sample data. If the *derived* model classifies the *hold-out* sample companies with a sufficiently high degree of accuracy the model is usually deemed to be satisfactory. It is deemed to have been replicated. If not, the procedure is recommenced until a model which *fits* both sub-samples is found.

Despite its seeming consistency with scientific practice, the basis for this practice is unsound. The process is pseudo-scientific. It is pseudo-scientific in that researchers usually systematically work through a series of derived stepwise MDA models testing each against the *hold-out* sample. Models which do not *fit* both sub-samples are rejected. Models which do fit are obviously retained and frequently published as being successful.

Replication in this manner denies the underlying motivation of testing in the so-called real world, i.e., to measure the extent to which hypotheses and research findings are able to be objectively tested evaluated. The pseudo-scientific approach of massaging the sub-samples of the data until researchers in the field of corporate distress classification actually obtains a function which *fits* both sets of data almost has an air of inevitability about it. Given enough variables and given the sample sizes commonly used in this type of research, if one tries hard or long enough we are almost bound to obtain something that works across both sub-samples, even by chance. Altman, himself, acknowledges this possibility when he talks of the possibility of *"bias due to intensive searching"*, [1968; p.600].

The extent to which individual ratios vary has a significant influence on the number of possible models that can be obtained from any sample of companies. As the variances of ratios increase there is a tendency for sub-samples to be relatively different from each other. In selecting from sample with high sample variances, it is much more likely that any sub-sample drawn will be significantly different from other possible samples, and thus markedly increase the likelihood of obtaining a *sub-sample specific model*. If this is the case, then we have to ask which model is the correct one. The number of possible solutions or models that can be developed from even small samples with this type of research method is vast.

If, for example, a random sample of 50 failed and 50 non-failed companies, each of which was measured by the same 20 ratios, was to be analysed by the sub-sample from which the MDA model is derived and the *hold-out* validation method, the distribution of the number of possible sub-samples of failed and non-failed companies, together with the number of possible combinations of ratios is reflected in table 5:1.

Table 5:1 The Number of Possible Samples from 50 companies

Number in the Sub-sample	Number of Sub-samples
1	50
2	2450
3	58800
4	921200
5	$1.06 * 10^{7}$
6	$9.53 * 10^{7}$
7	$6.99 * 10^{8}$
8	$4.30 * 10^{9}$
9	$2.25 * 10^{10}$
10	$1.03 * 10^{11}$
11	$4.11 * 10^{11}$
12	$1.47 * 10^{12}$
13	$4.61 * 10^{12}$
14	$1.31 * 10^{13}$
15	$3.38 * 10^{13}$
16	$7.88 * 10^{13}$
17	$1.67 * 10^{14}$
18	$3.25 * 10^{14}$
19	$5.78 * 10^{14}$
20	$9.43 * 10^{14}$
21	$1.41 * 10^{15}$
22	$1.95 * 10^{15}$
23	$2.48 * 10^{15}$
24	$2.92 * 10^{15}$
25	$3.16 * 10^{15}$

The number of combinations of 30 companies for example = 50 - 30 = 20 as the distribution is symmetrical.

Now examine the number of possible combinations of 20 ratios in table 5:2.

Table 5:2 The Number of Combinations of 20 Ratios.

Number of Ratios	Number of Possible Combinations
1	20
2	190
3	1140
4	4845
5	15504
6	38760
7	77520
8	125970
9	167960
10	184756
11	167960
12	125970
13	77520
14	38760
15	15504
16	4845
17	1140
18	190
19	20
20	1

So now if we take the products of these combinations in the following manner we will derive the number of possible combinations of sub-samples of failed, non-failed and combinations of ratios that are possible from an original set of 50 failed and 50 non-failed companies measured by way of 20 ratios. For example if we select 22 failed, 30 non-failed companies and 15 ratios we will have: $1.95^{15} * 1.41^{15} * 15504 = 4.26 * 10^{34}$ possible sub-samples from which to derive the MDA model of corporate distress. This would then be tested against the *hold-out* sample. Which sub-sample is the one likely to be most representative of the original?

The effective number of possible combinations of sub-samples will decline under two conditions. Firstly as the variance of each of the ratios declines there will be little apparent difference in selecting one particular sample or another. This, however, appears to be unlikely in practice. The larger the variances in the ratios, the more likely the sub-samples selected will produce MDA models that are sample specific. Conversely, where the variance of the ratios is relatively high, the significance of selecting one sub-sample rather than another may have a marked influence on the derived discriminant model.

Secondly, if step-wise discriminant procedures are used this will reduce the number of possible models to the number of ratios because the technique will optimise within any set of *n* ratios. The overall number of combinations will still remain very large.

As a methodology, the use of an *hold-out* sample to validate a derived MDA model is inadequate for two reasons. Firstly, it is at best a pseudo-scientific process because iterative procedures are almost always employed until a model that *fits* both sub-samples is obtained. This does not accord with the commonly held view of scientific replication. Secondly, this process, if carried out across all possible sub-samples would be extremely inefficient, to say the least. If, with the above example of 50 failed, 50 non-failed companies and 20 ratios, it took the computer one minute to calculate, then a computer would take $4.9 * 10^{27}$ years; an impossible amount of time which indicates the futility of using the method. Unless all possible sub-samples are investigated how can we be sure that the most appropriate MDA model has been identified. Furthermore, even if we do calculate all possible combinations, then by what criterion or criteria would we decide to choose one model rather than another? The sub-sampling of an original sample is inefficient in another respect also.

<u>THE INEFFICIENCY OF HOLD-OUT SAMPLES:</u>

The method is also inefficient in that in reducing the sample size in order to derive the model we unnecessarily increase the standard errors involved. Samples selected on a proper random sample basis, on average, will have the smallest amount of sampling error for that sample size. To reduce the sample size further in order to estimate an MDA model introduces a second dimension of errors. Even if the sub-sample is representative of the total sample, its use merely increases the magnitude of the errors markedly.

As discussed in chapter four, the importance of establishing unbiased estimates of the population mean ratios in discriminant research is paramount. The failure to do so will simply result in models which will not replicate in an *ex post* environment, let alone validate in a *ex ante* sense. The *hold-out* sample method of deriving an MDA model is highly inefficient in the sense that it leads to very much poorer estimates of the population means.

The standard error of the mean, that is the extent to which it will vary due to sampling error, is a function of both the sample size and the standard deviation in the ratio of the population as a whole. Now if for the time-being we assume that the population is infinite, we can estimate the increased error that is likely to occur as a result of using the *hold-out* sample method.

Now given that the standard error of the mean for random samples selected from infinite populations is given by the formula S_e [The standard error of the mean] = $S/n_1^{1/2}$ where n_1 is the sample size of either failed or non-failed companies.

If, for example, a random sample of the following dimensions is chosen:

n = 100 companies [50 failed and 50 non-failed] and S [the sample standard deviation of a ratio] = 3.00 we can examine the calculations precisely.

Now if the full sample of companies were used in deriving the MDA model the standard error of the mean would be $3/50^{1/2}$, that is, 0.42. This means that the true population mean would lie somewhere between the sample mean plus or minus 0.83 at the 95% confidence level.

Now if in order to develop a MDA model, we halve the sample to provide for a *hold-out* sample, the resulting standard error of the mean would equal $3/50^{½}$ or 0.6. This means that our best estimate of the population mean at the 95% level would lie somewhere between the sample mean plus or minus 1.176 at the 95% confidence level. As we subdivide our already small samples into two, we increase the size of the standard errors associated with the means. In this case we increased the sampling error by 41%. This is of particular importance because unbiased estimates of the population means are of critical importance to the developing of valid and replicable MDA models of corporate distress.

Now we must generalise this case for random samples from infinite populations. Given that the percentage increase in the standard error increases with the reduction in the sub-sample size it is defined as:

[[Standard error of sub-sample - standard error of sample]/[Standard error of the sample] - 1.] *100

This becomes

$$\left\{\left[\frac{S/[n/a]^{1/2} - S/n^{1/2}}{S/n^{1/2}}\right]\right\} * 100$$

where "a" = the reciprocal of the percentage of the sample used in deriving the particular model.

This becomes:

$$\left\{\left[\frac{S/[n/a]^{1/2}}{S/n^{1/2}}\right]-1\right\} * 100$$

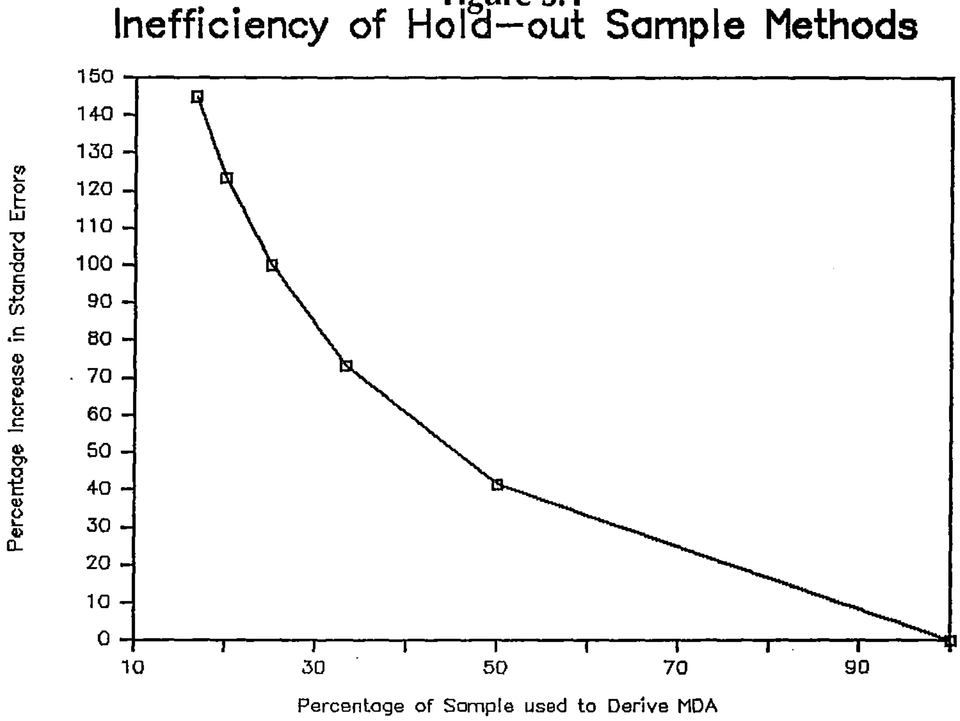
$$= [n^{1/2}/[n/a]^{1/2} - 1] *100$$

$$= \{[n/n/a]^{1/2} - 1\}*100$$

$$= [a^{1/2} - 1]*100$$

The percentage increase in the standard error of the mean is a constant for any particular sub-sampling proportion. As the proportion of the total sample used to compute the discriminant model reduces, so the standard error of the mean increases. It is not a function of the standard deviation, nor is it a function of the sample size itself. It appears to be a function of this factor alone. Figure 5:1 reflects this situation quite clearly.

Figure 5:1



Clearly as we decrease the sub-sample size for the purpose of deriving a MDA model to be tested on a *hold-out* sample we introduce an unnecessary source of error in the process. If, for example, we use 25% of the total sample of non-failed companies to derive the model, we will increase the standard errors by 100%. If instead we halve the sample, we will increase the standard errors by 41%. As shown in chapter four MDA models are driven primarily off means and mean differences and are extremely sensitive to even small changes in the mean ratios, then it is difficult, if not impossible to justify the further increasing of sampling errors in our estimates of the mean ratios by using the *hold-out* sample method. In fact, I will assert that there can be no justification for this pseudo-scientific and inefficient procedure at all. However, before closing the chapter on the topic it would seem to be necessary to introduce another two sources of realism to the situation.

NOW LOOKING AT THE REAL WORLD:

In fact, much of the MDA research into corporate distress has been plagued by an alarming shortage of failed companies. It seems more than a little strange that in order that we might predict corporate collapses more effectively, we need a lot more to collapse so that the quality of the predictions might be improved. Be that as it may, researchers have had to make do with the data that they can obtain. In the case of collapsing companies it has more often than not been necessary to use population data, that is, the complete set of failed companies. The ramifications of this has to be explored. Secondly, where either populations of failed or non-failed companies have been sampled instead, then the standard errors of the estimates of the means of the ratios involved need to be modified by the finite population correction factor. Firstly let us examine the situation in which researchers either have or are tempted to use a sub-sample of the population of failed companies from which to derive a MDA model before testing upon an *hold-out* sample.

In population data non-sampling error may exist, for example, measurement error, but there will be no error due to sampling at all. This is the case, by definition, for we have selected the population. If we then sub-divide the population of failed companies into two samples we inevitably introduce sampling error. There is no point in doing this at all. There can be no justification for making the estimates of the mean ratios less accurate than completely accurate for the pseudo-scientific notion of replication. Replication is not necessary when we can say something about the population itself. It is banal to attempt to do so.

Although the concept of the finite population factor [FPCF] is a fairly straight forward idea well discussed in most first year university level statistics courses, introducing the finite population correction factor where the population size of non-failed companies is usually small, is moderately complex, but nevertheless important. The FPCF is given by $[(N - n)/(N - 1)]^{1/2}$, [Newbold p:240], where N is the population size.

The percentage loss in efficiency, or percentage increase in the standard error of the mean ratio then becomes:

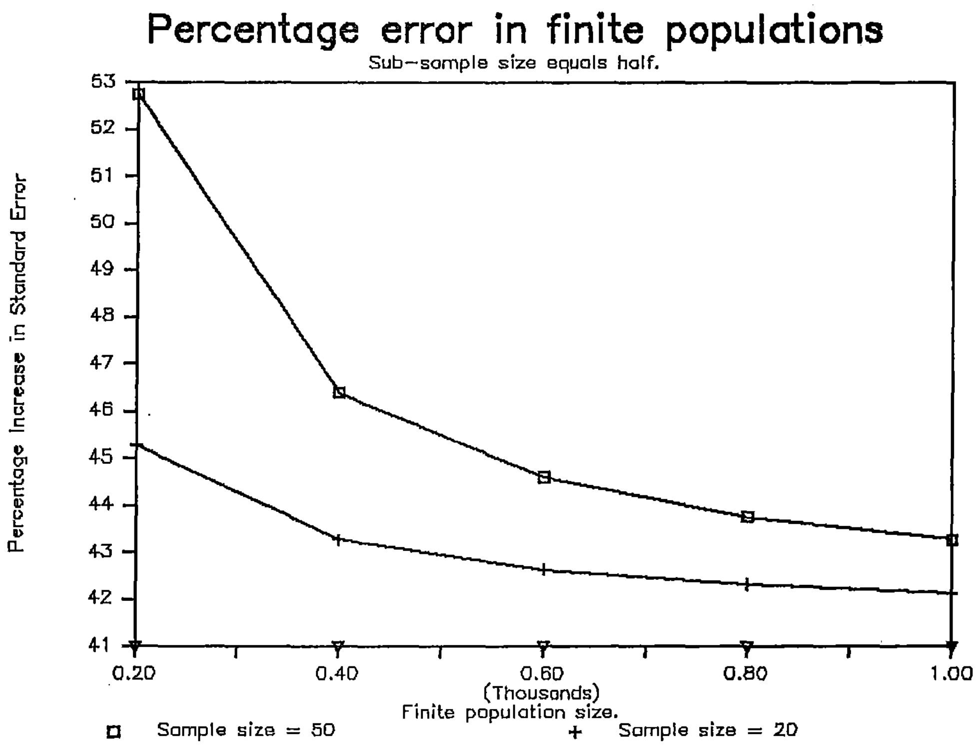
$$\left\{ \left[\frac{S/[n/a]^{1/2} * [(N - n/a)/(N - 1)]^{1/2}}{S/n^{1/2} * [(N - n)/(N - 1)]^{1/2}} \right] -1 \right\} * 100$$

which, for a sample divided in half, reduces to [See Appendix 5:1]

$$\{[a(N-n/a)/(N - n)]^{½} - 1\} *100$$

The percentage error introduced by subsampling a finite population is a function of the proportion of the of failed companies that is used to derive the MDA model, the population size and the sample size. With larger populations, although not infinite, the **additional** source of error introduced by the finite population is not significant. Frequently we are not dealing with large populations however. Figure 5:2 shows the extent to which the magnitude of the increased error in finite populations converges with the amount of error where the population can be regarded as being infinite:

Figure 5:2



There seem to be several points at issue here. Where the original sample of failed or non-failed companies is halved, for example, the increase in the standard error of the mean ratio will be 41%, i.e., $[2^{1/2} - 1]$ *100 for infinite populations. Figure 5.2 introduces two other variables. Firstly, as the population size increases, the standard error of the mean reduces and converges towards the level of increased error for infinite populations, i.e., 41% shown on the bottom line. Secondly, although the standard error of the mean increases with smaller samples, the amount of increased error that occurs as a result of halving the sample tends to reduce with smaller samples.

LIMITATIONS OF THE STUDY:

This analysis may be weaker than necessary as a result of treating the matter of sampling errors in a univariate sense. At this stage the conclusions would seem to be valid and I believe them to be so, but more research on the subject might strengthen the notion that a multivariate approach might be more appropriate. Even if this is the case, it would be more than difficult to escape the underlying argument.

CONCLUSION:

In all cases, the process of sub-sampling the original sample in order to develop a MDA model before testing it on a *hold-out* sample increases the size of the sampling errors involved. There can be no justification for this. Obtaining the best estimates of the population mean ratios for companies about which we might wish to say something significant is fundamental to the MDA modelling process. It has already been shown how sensitive that the MDA technique is to small differences in means. The data is frequently scarce enough without wasting it by ensuring that our estimates are even less accurate than when we started. The *hold-out* sample approach is not only pseudo-scientific at best, it is also less efficient than using the total sample to derive the model.

Chapter Six

THE PROBLEM OF MISSPECIFICATION.

INTRODUCTION:

A major problem with all multivariate modelling, whether it be discriminant analysis, multiple regression or another technique is that important variables may be left out of the equation. This means that the model is incomplete, or in the parlance of the econometrician, it is misspecified. Misspecification distorts the relative contribution of all the variables involved.

MODEL SPECIFICATION:

Although it will not be an easy process, the main thrust in the building of multivariate models of corporate distress should be towards the correct specification of the particular model. The debate in this area of model building has long been either about predictive validity or adequate specification of causal relationships. Although the former approach seems to have produced some exciting short-term results over the last two decades, it has failed to achieve satisfactory long-term results. After two decades of data dredging, in which large quantities of data on small samples of companies has almost literally been poured into computerised MDA programmes, little of an enduring quality remains. I suggest that it is time to pursue the causal path of enquiry. Before we proceed too far however, a cautionary note should be recognised. *"Most writers seem to agree that an economic theory is useful to provide an initial specification, realising that this theory will almost certainly be insufficient for a complete, dynamic specification, with some missing or poorly measured variables,"* [Granger, 1990; p.18]. Perfect, or complete specification of economic models is probably close to impossible. This, I suspect, is likely to be true in the field of corporate distress forecasting, for our integrated theoretical base is extremely thin, to say the least. Most of the MDA research into classifying and forecasting has been of the data dredging type. As a result it would probably be fair to say that all of the published models have been misspecified, particularly in relation to a macroeconomic context.

When writing on the subject over two decades ago, Kendall [1967] provided a useful cautionary note. *"The systems we have to study are far too complicated to permit anything of the kind [i.e., perfect specification] and we shall find, I think, that we are continually having to curb an ambition which tempts us to build models of too great a generality,"* [p.2]. He further counsels that *"model building should start with simple and modest models, and work towards the more complicated systems by integration, rather than start with attempts at comprehensive models,"* [p.3]. Kendall's comments are indeed relevant, yet after over two decades of almost blind data dredging we need to move towards a more satisfactory level of multivariate model specification in corporate distress classification and prediction.

Mirer [1988] provides a useful discussion on the problems of misspecified models. He states that *"... it is difficult to decide which variables should be included in the model, and it is likely that some models we see or create will be misspecified,"* [p.140]. Although he was talking about regression in particular, the same arguments apply to MDA modelling for there is a perfect correlation between discriminant z-scores and binary regression dependent variable estimates when both are applied to the same set of data. Mirer continues to point out that we are in continual danger of making two types of mistakes in specification. *"We might* exclude *a relevant variable from the regression, or we might* include *an irrelevant one,"* [p.140]. Excluding a relevant variable will change the *"true"* coefficients of every other variable in the model so that our multivariate relationships are distorted in relation to the way that they actually operate. Furthermore the *"inclusion of the irrelevant variable can lead to the misinterpretation of the true economic process,"* [p.141]. This may also produce a confusion between correlation and cause.

One of the major problems in the MDA modelling of corporate distress is that the research community appears to be far from being able to produce a concensus about a satisfactorily specified model, i.e., there is no commonly accepted theoretical framework. So far about 100 ratios have been investigated, cash flow variables have been vigorously pursued, non-accounting variables have been put forward with varying degrees of enthusiasm, [e.g. Argenti] and macroeconomic variables have been suggested, [e.g. Foster]. Each of these suggestions will no doubt have a particular validity, yet we still have no concensus on specification. Bearing in mind the cautionary note sounded by Kendall, what we need is a correct specification of the model of corporate distress.

As it is not possible to pursue all of the issues involved in this process, I will focus on ways in which we might be able to improve the specification of our model of distress. These suggestions are not put forward as a panacea of all of the problems in this regard, but that progress might be made. These issues involve the use of macroeconomic variables, the use of lagged variables, and finally the potential of a first differences or change approach. Each will be examined in turn. Before examining these possibilities, a more generalised strategic approach, **the top-down approach** to investment analysis, should provide a more appropriate methodology by which we might establish a framework for an improved model specification.

THE TOP-DOWN APPROACH TO FUNDAMENTAL ANALYSIS:

For many decades now, practising investment and security analysts have recommended that investors should take a systematic approach in the search for sound investments. This systematic approach involves three stages. Firstly, they have suggested that the wider economy should be evaluated to ascertain the extent to which the current economic climate and short-term outlook is suitable for investing. Secondly, markets and industries should be evaluated in the context of a suitable economic outlook. Finally, having satisfied themselves that they have identified a suitable industry or industries, investors should carry out the kinds of *fundamental* analysis of financial statements that is commonly recommended in standard accounting and finance textbooks.

Academic investment analysts [e.g. Jones,1990; Reilly,1989] have subsequently adopted this analytical framework as being rational. In chapter nine *"The Process and Theory of Valuation"*, Reilly, [1989], outlines his contention that *"the discussion should first center on the analysis of aggregate economies and over all securities markets. Only after this is done can different industries be considered from a global perspective. Finally, following the industry analysis, you should consider the securities issued by various firms within the better industries,"* [p:306].

The top-down approach requires that an industry context be established. Most of the MDA models of corporate distress have attempted to reflect this point and have generally not been generalised models but industry models. As this is a well recognised facet of the modelling process the matter will not be re-examined in this thesis.

In brief, a company does not stand alone as an independent entity. It stands within an industry and within an overall economic context. It is almost remarkable that this kind of approach has not already been applied to the MDA modelling of corporate distress. It is, of course, not an easy kind of analysis to carry out as even a cursory reading of writers like Reilly and Jones will reveal. Despite this, it has a simple, but not simplistic, logic about it. Rose, Andrews and Giroux [discussed fully later in this chapter] clearly demonstrated that the quarterly level of corporate collapse in the United States of America relates very highly to macroeconomic variables. This combined with the simple rationale of the top-down approach to analysis, I believe, should lead us towards a more appropriately specified model. This model commences with the macroeconomic context.

I: THE USE OF A MACROECONOMIC CONTEXT:

All of the multiple discriminant models of corporate failure published to date have been microeconomic models. Although two regression studies of the relationship between macroeconomic variables and the quarterly level of corporate failure have been carried out by Altman [1983] and Rose, Andrews and Giroux [1982], there have been no such variables incorporated into MDA studies. Although Foster [1986] has suggested that there is a need to pursue such investigations, it seems strange that while standard accounting textbooks [e.g.Gibson, 1989; p.123] frequently recommend that company ratios to be interpreted in the light of industry norms or averages, researchers have not included any macroeconomic variables in order to provide a total environmental context.

Altman and Macroeconomic Variables:

Altman [1983] summarizes his earlier publications in *Business Week* which indicate a relationship between the aggregate level corporate failures and five macroeconomic variables which he calls, economic growth activity, credit availability or money market activity, capital market activity, business population characteristics and price level changes. Each of these concepts require some further explanation.

Economic Growth: *"sales and earnings of individual enterprises are directly related to overall business activity....we should expect a negative correlation between series which reflect the nation's economic health and business failures," [p.85].*

Money Market and Credit Conditions: *"One of the more lively current economic debates concerns the effect that the nation's monetary stock has on our economic conditions...credit availability and its cost. most certainly does matter....we can expect that the propensity to fail will be increased during periods of relatively tight credit conditions vis-a-vis periods of easy credit,"* [p.86]

Investor Expectations: *"The relationship between common stock prices and business failures is predicated on both empirical and theoretical grounds......In order to reflect overall stock market performance, we chose the change in the Standard & Poor (S&P) index of stock prices." [p.*87].

Business Population Statistics: *"..when we observe the frequency distribution of failures with respect to the age of the firm, aggregate data show quite clearly that over one-half of all failures occur within a firm's first five years and almost one-third within three years," [p.88]*

Price Level Changes: *"...increases in prices, especially unanticipated increases, tend to be inversely correlated with failure rates,"* [p.90].

The Econometric Model:

Altman produced a complicated regression model to forecast the level of company failures. *"The period 1951 through 1978 was chosen, and a set of explanatory variables reflecting various macroecomonic pressures was examined within a first difference, distributed lag regression structure. Findings indicate that a firm's propensity to fail is heightened due to the cumulative effects of reduced (1) real economic growth, (2) stock market performance, (3) money supply growth, and (business formation),"* [p.98]. Despite this reassuring statement however the adjusted R^2 was only an unconvincing 0.26. His conclusion however is more important. *"Where macroeconomic expectations could be extremely important is in the choice of appropriate prior probabilities of failure. This level is useful in adjusting the optimum cutoff score for micro bankruptcy prediction models,"* [p.98]. The point has been made before but at least Altman attempts to progress down necessary channels of research.

Despite what might be called a valiant effort to research the question, Altman fails to grapple with the problem satisfactorily in two ways. Firstly, Altman understates the case for relating company failure to macroeconomic variables. Not only does the level of company collapse affect the cut-off point by way of the *a priori* probabilities but the macroeconomic context may well radically alter our interpretation of a particular set of ratios. A *high* debt ratio, for example, may be viewed favourably in a booming economic environment which is likely to remain stable for some time. On the other hand the perception would no doubt be markedly different in less buoyant times or when the economic outlook is less favourable to the fortunes of a particular industry.

Secondly Altman does not actually prove his case very well at all. With an adjusted-R^2 of 0.26 he provides little evidence that the level of company failure in the United States actually does relate to the economy. If corporate collapse actually does relate to economic circumstances then Altman has clearly not identified the important variables or the appropriate statistical model. Clearly his model is misspecified.

Finally, the level of company collapse, as a percentage of the total population of companies over time could actually be held to be so small that any variation of the cut-off points that would arise as a result of changing *a priori* probabilities would be so small that it could hardly be considered to be material. Altman's United States findings for the period 1950 - 1981 were that the level varied from a low of about 25 per 10,000 in 1977 - 78 to a high of nearly 70 collapsing companies per 10,000 in 1961 - 62. When translated into percentage terms these variations in *a priori* probabilities could hardly be held to relate importantly to significant variations in cut-off points in discriminant models. If on the other hand the failure rates varied more dramatically in particular industries, the issue might be of greater importance.

Rose, Andrews and Giroux and Macroeconomic Variables:

In a most important piece of research, Rose, Andrews and Giroux [1982] have established a much more satisfactory relationship between economic indicators and business failure. They employed macroeconomic variables and developed a quarterly six variable multiple regression model using data from the 1960 to 1980 period. The resultant R^2 of 0.912 is highly satisfactory and provides adequate validity of the model's specification.

Their basic thesis is found in the statement, *"To the extent that business failures are linked to the cycle, the variables suggested by the business cycle theory should be correlated with indices of business failure,"* [p.23]. They argued that as *"virtually all major theories fall into three groups:*

(1) Supply or cost-push theories;

(2) Monetary theories; and

(3) Savings-investment theories. [p.23],

then these should be used to identify the relevant macroeconomic variables. They also added a list of nine *"Leading and Coincident Indicators"* such as the S&P 500 composite stock price index, the Dow Jones industrial average, and the unemployment rate. As a result they identified a total of 28 macroeconomic variables for analysis. These 28 variables were bivariately correlated in order to identify those that intercorrelated above the 0.80 level. Those with high intercorrelations were discarded, presumably to reduce multicollinearity.

The data was analysed by way of a forward stepwise regression with lagged variables. The model finally selected was one containing six variables representing each of the three classes of business cycle indicators listed above. Four of the indicators had a lagged relationship to the level of corporate failure of two to four quarters and the other two had no significant lagged relationship in the model.

Rose et.al., have shown quite usefully, that despite doubts about the importance of the small numbers involved, certain macroeconomic indicators can be used to forecast the level of business failure. Although this is both useful in that *a priori* levels might be used to establish cut-off points for discriminant models, the actual variation is probably too small to have any significant influence (unless there are only quite minor mean differences between the two groups of course). What is needed is a method by which discriminant models of corporate failure might incorporate macroeconomic variables. This would enable z-score methods to be used by those with an interest in evaluating the extent to which an individual company was likely to collapse. Rose, Andrews and Giroux have provided convincing progress in this respect. They have not solved the problem however, for we still do not appear to be able to build reasonably enduring MDA models of corporate distress.

Market and Industry Effects:

Another group of studies may provide some of the building materials for a much more comprehensive approach to the forecasting of corporate distress. These studies were initiated by King [1966] and followed up by several other researchers.

King carried out a factor analytic study of companies on the New York Stock Exchange over the period 1927 to 1960. Although he was investigating share price movements, rather than company collapses, his findings may have some relevance to the future direction of the study of forecasting company distress. He identified what he called a *market* factor, and *industry* factor and a *unique* component. His confirmatory cluster analysis indicated that 52% of the variation in the share prices could be attributed to a *market* factor and a further 10% to a *industry* factor. Meyers [1973] followed this work up and confirmed the importance of both factors, although there was some weakening of the industry factor in a later period. Other researchers [e.g. Blume [1971] and Livingston [1977] have also confirmed these results in separate studies.

Although King et.al., have showed that both a market and an industry factor seems to exist, and although Rose et.al., have shown that there is a strong relationship between macroeconomic variables and the incidence of corporate collapse, research in this field has not provided a methodology by which we can build macroeconomic variables into distress prediction models. The rest of this chapter will therefore focus on several possible methods by which this might be achieved.

Identifying Macroeconomic Indicators.

Identifying the type of macroeconomic indicators that might be usefully incorporated into MDA models is a major research problem in itself. Here I wish to explore some of the possibilities.

Forecasting or classifying corporate failure more satisfactorily would then require us to include the appropriate macroeconomic variables that comprehensively reflect the economic situation. This is a major problem in itself for it would appear that we have no single variable that can encapsulate the economic conditions of the time. Here we need to integrate business cycle theory with corporate distress theory. The proponents of the top-down approach to investment analysis no doubt wonder, I suspect, why this has not been embraced already. One approach to this problem might be to use a array of variables, such as those developed by Rose, et.al. If these macroeconomic variables were factor analysed in order to identify the underlying factor structures, this would provide a linear transformation that might enable us to establish a single composite variable that would provide a barometric indication of the state of the economy for each year.

Associated with the problem of identifying a satisfactory measure of the state of an economy, before relating this to particular businesses, is the question of forecasting the composite variable itself. If individual company ratios are to be used to forecast whether companies are likely to collapse in a forth-coming period , then we will require a model to forecast the likely state of the economy for the same period. A z-score would then have to be interpreted variously depending upon the forecasted state of the economy itself. If it is customary to argue that an industry context should be evaluated, then why not a macroeconomic context? The microeconomic MDA models published to date are too simplistic if they do not include this dimension. Clearly the more traditional approach misspecifies the equation.

Modelling with Macroeconomic Indicators.

Having identified a macroeconomic factor it should be possible to classify the state of the economy by way of an expansionary condition or outlook, a stable or equilibrium condition or outlook, or finally a recessionary condition or outlook. In short, it would seem that it is necessary to relate financial performance of companies to each of the stages in the business cycle. Again, we seem to be hampered here by a lack of a suitable theoretical framework for evaluating financial ratios. It might be necessary to develop one in the process.

During the first type of economic environment, companies will generally be facing expansionary conditions under which all of the facets of a growth economy are enjoyed. During this period we would expect managements' attitudes and outlooks to be reasonably buoyant, their investment and borrowing strategies to be expansionary, and even their credit control to be less strictly managed. Under these conditions we would expect companies to have markedly different ratios than they would during a contracting economic period. As already suggested, the mean debt ratio might conceivably be higher than in during a recessionary period. This hypothesis needs to be evaluated, of course, but the suggestion has a plausible, if not logical appeal. Furthermore, as all industries do not prosper during the same or similar economic conditions, industry factors will need to be taken into account.

Similarly, during a period of economic slow-down, business confidence would be expected to be reflected in a tailing off of the growth rate in sales, at the same time managements' investment plans might be expected to be somewhat dampened, and there are likely to be signs of tighter managerial control being imposed. Management might be expected to pay more attention to the collection of receivables, might be expected to take firmer control over inventories and might be expected to look towards reducing levels of debt. There are many more aspects in which these and other factors would be reflected in the financial reports and the associated ratios.

Finally, under conditions of recession, managements might be expected to be looking towards radically reducing levels of debt, certainly imposing much stricter credit standards and indeed it might well reflect the weakness of a company if this has not already been carried out by the time a recession arrives. During a recessionary period of the business cycle, we would expect different mean ratios for a wide range financial measures.

If this kind of trichotomy of the economy could be established, and there do not seem to be any impediments to measuring such a phenomenon, then the mean ratios, or other variables, would be much more likely to remain stable during similar economic periods. The extent to which mean industry ratios for failed and non-failed companies relates to macroeconomic variables is hypothesized here and would have to be evaluated empirically, but the suggestion provides a reasonable starting point. It is difficult to imagine alternative relationships. The usage of ratios over many years, during which there are several different types economic environments will inevitably lead to the collapse of MDA models of corporate distress because the respective group mean ratios are extremely unlikely to remain stable. There is no theoretical reason for the mean ratios to stay the same during such periods of different business conditions.

This methodology is most unlikely to provide a panacea of all of the problems associated with developing stable financial accounting measures which will provide the same sort of information signal to researchers under all conditions, but the current practice of combining all data from diverse economic environments has proven itself to be a failure. The multiple linear discriminant method simply cannot cope with situations in which a simple debt ratio of 78%, for example, means that management has borrowed wisely in order to capitalise on expanding opportunities in one economic environment, yet in another, reflects over-ambitious, irresponsible or even reckless managerial behaviour. By relating the variable to the economic conditions in some way, i.e., relating it to the stage in the business cycle, we may have a better chance of success.

A Three Function Model:

A three function model then would seem to be warranted. In order to test this approach we would probably have to trichotomise the economic outlook for each of the forthcoming years according to the methods suggested above. We would then need to group the data from each of the preceding years into three sets according to the outlook or economic conditions. In this way we would have three sets of data which could then be analysed by way of multiple discriminant analysis. In short, we would require three models, each relating to the three stages of the business cycle because many of the mean ratios [or industry norms] could be expected to be different under various economic conditions. This approach will certainly not solve all of the problems in the modelling of corporate collapse, but it has a rationale that relates to a logical framework. It relates microeconomic performance of individual companies to business cycle theory.

Such a model might commence with a factor analytic or principal component regression based upon the ideas discussed earlier. This may enable us to forecast the stage in the business cycle. This, of course, will burden us with yet another set of forecasting problems but the literature is at least well developed on the subject.

Subsequent to the development of such a model of the business cycle, attention could then be paid to the microeconomic aspects.

MDA models might then be developed for each stage in the business cycle. Our ability to do this will be a function of our being able to specify appropriate models. It will also be a function of the extent to which there are significant mean differences between failed and non-failed companies, and the extent to which the mean ratios might be stable across the years that are characterised by the same economic conditions. If we cannot effectively relate particular mean ratios to the state of the economy, it is doubtful whether this approach will allow us to progress far. It is, I believe, logical and provides a reasonable hypothetical framework for further research. All of the relationships, however, will not have the same timing impact. Some of the relationships are likely to be lagged.

II: THE DEVELOPMENT OF A SET OF LAGGED RELATIONSHIPS:

Although several studies have evaluated the extent to which various models have been able to predict corporate collapse, they have simultaneously included all of the ratios and other variables in the same time period. This provides a rather flat, uniform and unrealistic kind of analysis. Beaver showed that certain ratios tended to indicate impending failure several time periods ahead. Macroeconomic research has long used leading, lagging and coincident indicators of various economic conditions. It is unrealistic for us to expect that all of our ratio indicators will have the same time relationship. The leading indicators will almost invariably have different, or even distributed lagged relationships with the incidence of collapse. Rose, Andrews and Giroux showed this to be the case within their forecasting model and it is to be expected that this will also be the case with financial indicators. It would be more surprising if this were not so. This would require a more realistic specification with the inclusion of lagged and possibly distributed lagged relationships between the ratios and corporate fortunes. This probably does not make the search for a properly specified MDA model of corporate collapse any easier, but the inclusion of such an approach is much more likely to allow researchers to reduce the likelihood of specification errors. Not only are lagged relationships likely to exist, but the dynamics of change in a company's financial condition need to be investigated.

III: THE DEVELOPMENT OF A CONTEXT OF CHANGE:

A major problem with the majority of the corporate distress studies carried out to date is that they have tended to be static analyses. That is, they tend to reflect the financial conditions of the sampled companies at a single point in time, [even those developed from the income statement] while the companies themselves are more usually in a state of flux or change. The dynamics of company performance have not yet been effectively taken into account in the modelling process.

One of the commonly used methods of analysing company financial performance is that of standardising the figures in such a way that it produces a *"common-sized"* balance sheet and income statement. Balance sheets can be standardised in a *vertical* manner by expressing all values as a percentage of the company's total assets. Similarly, income statements can be standardised in the same vertical manner by expressing all values as a percentage of total sales. This approach allows us to compare companies and gain insights that would otherwise be difficult to make. This approach also produces many of the currently and commonly used ratios. Although useful, this form of standardising does not add a dimension of change however.

If on the other hand, the financial statements are *horizontally* standardised, the elements of the balance sheets and the income statements will be expressed as a percentage of the corresponding previous year's figures. Under this method, the total sales revenue, for example, would be expressed as a percentage of the previous year's sales and thus provide a very succinct expression of the trend in the business. Although less commonly used than the vertical method of standardising, this approach provides a simple but effective way of measuring the extent to which a company's financial circumstances are changing. By incorporating this method into the MDA modelling of corporate distress we would be doing more than merely adding to the potential number of ratios. We would be adding a dimension of change into an otherwise static analysis. The dynamics of the financial situation that are likely to provide an improvement in the specification of corporate distress models.

A second method by which the dynamics of company performance might be taken into account is in the use of a *first differences* approach. This would involve not just recording the value of a particular ratio itself, but also the change in that ratio over the previous period. The level of the debt ratio, for example, is one single piece of information that might be useful in itself. It may partially reflect the vulnerability of the company, yet the direction and the magnitude of the change in that ratio, in the context of a change in the business cycle might allow us to more fully reflect the financial circumstances of the companies involved. In other words, a single ratio such as the debt ratio, provides us with too little information. Its information value may be enhanced by measuring the change in the ratio. This kind of trend analysis, taken in the light of forthcoming economic conditions would provide a more comprehensive specification of a model of corporate distress.

CONCLUSIONS:

In this chapter I have attempted to grapple with some aspects of the need to correctly specify our models of corporate distress. I am acutely aware that this is a very difficult task, but the MDA models which have been developed have been sadly lacking in this dimension. The misspecification of MDA models of corporate distress seriously undermine the value of the efforts made to develop sound classifying and forecasting methods. The development of a macroeconomic context, the inclusion of lagged variables and the inclusion of variables that reflect a more dynamic aspect of company performance seem to be important.

Chapter Seven

A RESULTANT MODEL-BUILDING STRATEGY.

INTRODUCTION:

It seems that after extensive criticism of the use and misuse of Fisher's multiple linear discriminant function over the last two decades, both from inside and outside the field of accounting and finance, there has been a developing need for a re-examination of some of these matters. This need arises because it appears that the relative importance of some of the issues now appears to have been over stated. This chapter draws together the essential elements of the relevant published research, adds the findings of this study and attempts to provide a methodological framework in such a way that researchers might be able to steer that difficult course between dealing with the realities of the kinds of accounting data that we are more or less obliged to use and unjustifiably and possibly fatally breaching the assumptions. If this can be done, we might have a better chance of developing models that will be more universally accepted than they have been so far. Before investigating particular strategies for handling the rather complex set of issues surrounding the use of linear discriminant models in corporate distress research, we need to briefly examine the reporting practices of the published research, for it would seem to me that in our drive for an economy of words in journal articles, there has been a glossing over of important and unresolved aspects of the modelling process, not so much in the summary articles [e.g. Jones 1986] but in the more empirical papers.

REPORTING STANDARDS:

Given the complexity of developing valid and durable MDA models, the statistical reporting practices of the published body of research on the topic has not been such that a rigourous examination of any particular researcher's findings is possible. Understandably, the editors of academic journals require an economy of words, but I suggest that there is a very definite need for more comprehensive reporting because multivariate research into classifying and forecasting corporate distress is still in its infancy. Given the wealth of criticisms in the literature over the last two decades, research developments will be more likely to occur if there is a more complete reporting of many aspects of particular studies. Financial accounting data is not renowned for being ideal for forecasting corporate collapses at the best of times, largely because of the diversity of accounting policies, and as argued elsewhere in this thesis, also because of the lack of stability in mean ratios over time. The relationships between the data seem to be varied and complex. As a result, a more complete reporting of the statistical analyses involved is required. This involves several facets.

Despite the fact that the MDA technique assumes the equality of the variance-covariance matrices, I can find few instances amongst published research where this was actually statistically tested, [e.g. Betts & Belhoul,1987; Taffler,1982]. There are conditions under which MDA techniques are extremely sensitive to this assumption, yet empirical researchers seem to be content with a less than satisfactory level of investigation and reporting. I would suggest that reported MDA research should include both of the sample variance - covariance matrices and the tests of the extent of the significance of the difference between the two. Despite more than 20 years of published research, the state of our knowledge is extremely limited. Other researchers are effectively left in the dark about very important aspects of the modelling process.

In fact, even a cursory enquiry into the matter will show that it is clear than we understand far too little of the interaction of ratios and other accounting variables. Foster [1986, possibly followed by Lev,1974] provides perhaps the best collection of reports into financial statement analysis, but this question is still in its infancy. Developments are much further advanced in the United States, of course, but very little is known about New Zealand data, [Firth & McLean,1987]. If MDA research into corporate distress reveals, as I suspect, that the variance - covariance matrices of failed and non-failed companies are significantly different, then we have not failed, but have identified a very important area for further research. Why then are the covariances between the financial variables of failed and non-failed companies significantly different? Although the question is clearly one that we cannot answer yet, a researched answer is very definitely required.

Again, the reporting practices are well below desirable standards needed for the further development of our understanding of the topic when it comes to evaluating the extent to which the data is multivariately normally distributed. Perhaps researchers are too keen to report ultimate solutions. Perhaps the issues have not been perceived as being important. Perhaps there is another explanation, but at best this failure is difficult to justify, particularly in an area of research in which we know so few of the answers. If we are in breach of the multivariate normality assumption, the extent of this should be statistically evaluated and reported in the published account. It would seem that researchers in the field of corporate distress too readily overlook the findings of researchers in the associated disciplines of multivariate statistics and econometrics. The evidence is that such matters are frequently important. In a way this tendency is understandable for many researchers using the MDA technique in accounting and finance do not appear to be well educated in the field of multivariate statistics, and indeed, to be fair, it is difficult to be well educated in both fields, but I believe that the quality of our research in this field depends upon it.

The kinds of reporting needed for the development of the field of multivariate modelling of corporate distress requires us to return to some elementary issues. Because of the difficulties of obtaining "good data", I would prefer to see the original data reported so that it can be available for scientific scrutiny. This might be asking too much of academic journals however. Because multivariate normality is almost always an issue, then the tests of significance of the extent to which the groups of failed and non-failed companies depart from this criterion should be reported. Because the question of the equality of the variance - covariance matrices is almost always an issue, then the actual sample matrices and the tests of the extent of the difference between the two groups should be published. Finally, because the significance of the difference between the mean vectors of ratios, [and indeed other variables if they are used], is perhaps the most critical issue, then the mean vectors and probably the univariate, if not the multivariate tests of the significance of the difference between the two should be reported.

The flow of research articles in the field of forecasting corporate distress has dried up somewhat in more recent years, possibly reflecting not so much that fact that the subject has been fathomed, but more the frustration of not being able to satisfactorily achieve the objective of classifying and predicting. Finally, perhaps it is time for a comprehensive collection of articles and source data to be made available to the international research community so that further progress might be made in the field. It is clear that we know so little about forecasting corporate distress from financial accounting data, if indeed, it is ultimately possible. Further progress will no doubt depend on research into other variables such as cash flows, macroeconomic indicators, strategic management decisions, as well as financial accounting reports, but the full reporting of the research findings, strengths and weaknesses, should be encouraged as part of the empirical process.

THE MOST CRITICAL ISSUES:

Combined research into the distress modelling process has resulted in many aspects being discussed widely in the literature. Non-statisticians wishing to use the MDA technique on their data could easily be forgiven for being confused. Not only is there a need for more complete reporting of research in this field, but there is also a need for a clear statement of **the most critical** methodological steps required. I suggest that there are six issues of vital importance. Although widely discussed in the literature as being of critical importance, I have tried to draw from the literature, and from the results of the experimentation carried out in this particular research, the notion that the those issues widely held to be critical may not be as important as suggested in earlier publications. While they are indeed important, they are of lesser importance. In order of significance these are; the selection of the variables involved i.e, a correctly specified model, the need for random sampling from well-defined populations, the efficient use of the sample data, the need for mean variables that are stable over time, significance of the difference between the respective group means on each of the variables used in the modelling process, and finally, intertemporal validation. Each of these will be discussed in turn.

I: Selection of the Variables.

The most important issue would seem to be that of the selection of the variables. Misspecification of models is a subject comprehensively addressed in the econometrics literature. Within the corporate distress prediction and classification literature the question of choosing appropriate ratios and other variables seems to be treated with less concern than it should be. If the wrong variables are chosen then the model will be misspecified. As yet there is little concensus about a corporate distress theory and as a result perhaps all models are in danger of being misspecified. Research into this aspect will no doubt involve a long arduous process before we are able to specify multivariate models satisfactorily.

II: Random Sampling.

Of all the violations of the assumptions in MDA research into corporate distress, I think that the failure to use proper random sampling methods is one of the least forgivable. The failure to use random samples, or at least stratified random sampling methods where this might be appropriate, means that after all of the work that might have gone into the development of a particular model of corporate distress, the best that we can say is that the results are sample specific. Researchers seeking to develop MDA models must pay more than cursory homage to this issue for it is vital to our ability to speak meaningfully about the parent population.

Jones [1987] points out that there still exists a bias in our sampling procedure when we attempt to collect samples of failed and non-failed companies because failed companies are most often found in the smaller unlisted companies. Size appears to be a significant variable and thus our sampling of listed companies tends to make the samples unrepresentative. This is a difficult problem to overcome. Most of the smaller companies that collapse are privately owned and the data is not generally publicly available.

Probably as an integral part of correct specification clear definition of the population about which we will eventually wish to speak. Unless we are extremely fortunate, without random sampling from an appropriate sampling frame, MDA models will at best be sample specific. Where the population data cannot be used, the first step in the MDA modelling process is therefore to define the respective populations precisely and to select at random from the group of failed and non-failed companies. This is a demanding process, and one that enthusiastic researchers might be inclined to pay scant attention, or be seduced away to somehow hand pick the companies (frequently called matching or matched pairs), but it must be done scientifically. No lesser standard can be accepted or the results will be biased and sample specific.

III: Using the Sample Data Efficiently and Effectively.

The third most critical issue in MDA model development is that of effectively and efficiently using the data that we have gathered. As already indicated in chapter five of this thesis, the findings of this research are completely dismissive of the use of *hold-out* sample methods. The main argument is that using such methods only adds confusion to an already difficult process. Data on failed companies is frequently so scarce and the sample sizes are often so small that the further sub-division of the sample into one from which the MDA model is derived and a *hold-out* sample only serves to increase the errors involved. I find it difficult, if not impossible, to find any virtues in the *hold-out* sample method. Of course, the abandoning of *hold-out* sample methods alone will not guarantee that models will proceed beyond sample specificity but model builders are more likely to be able to develop models that will endure the inevitable rigours of cross examination if they employ the best available sample company data in the model-building process. This will reduce the magnitude of the sampling errors involved.

IV: The Need for Stable Mean Vectors.

The fourth critical issue in the development of corporate distress models is that of obtaining mean ratios for different time periods for each of the failed and the non-failed groups of companies that are **stable** over time. This is probably by far the greatest stumbling-block to the eventual development of models that will validate both in an *ex poste* and *ex ante* manner. Although it is beyond the scope of this research to identify ratio data which might provide means that are stable over time, it is important to recognise that multiple discriminant analysis primarily works off means and mean differences. An early step in the MDA model-building process must be the search for such ratios that provide a consistent signal. The problem has always been that some mean ratios vary quite significantly [e.g: Lev 1974; Foster 1986] from year to year as economic factors, the mix of business decisions, financing decisions, operating decisions, financial reporting policies and the macroeconomic environment each vary over time. During some economic time periods particular industries experience varying fortunes which are reflected in the financial statistics. Unless the mean ratios remain stable over time, then it is very difficult to foresee a situation in which MDA models could ever validate intertemporarily. If there is a single obstacle to the successful use of MDA techniques in forecasting and classifying corporate distress, it is likely to be the aspect that stable group means for each of the variable involved are an absolute necessity.

The problem is that even when historical data seems to point in the direction of the stability of mean ratios for failed and non-failed companies, in projecting into the future it is not possible to know whether or not this phenomenon will continue to hold. Without this, the use of multivariate techniques, and possibly the use of accounting ratios, for prediction purposes appears to be doomed to failure.

Dambolena and Khoury [1980] used the standard deviation of the ratios as a measure of variability in their study and developed two discriminant models; one using these measures and the other without. They found that the model using the standard deviation classified significantly better. This may a step in the right direction, but it does not satisfy the need for stable mean ratios over time. Fisher's model was developed originally for scientific data whose measurements were unlikely to change. Any departure from this is likely to present what appear to be insurmountable problems for those of us using less stable measurements.

V: The Need for Significant Differences in Groups Means of the Variables Involved.

It would seem that in our somewhat blind search for ratios or data we may not have been clear about the kind of ratios or variables that will enable us to discriminate well. The fifth critical issue in the MDA modelling of corporate distress is that of the significance of the difference between the means of the variables between the failed and the non-failed companies.

Firstly, chapter two illustrated the fact that while it was accepted that *a priori* probabilities are important in the modelling process, the issue pales into insignificance compared with the need to have a significant difference between the means on the original variables.

Secondly, the question of the multivariate normality of the data, and the equality of the variance-covariance matrices may be of importance, yet the arguments put forward in chapter four together with earlier studies by researchers like Lachenbruch, and Gilbert, for example, support the notion that these issues may be less critical than the significance of the difference between the mean of the variables involved.

The concept of mean variables or ratios being significantly different embodies two interdependent ideas. The first is obviously that the means of the respective groups must be sufficiently separated. If the means are not sufficiently separated then we are effectively searching amongst fine differences and multivariate normality, the equality of the variance - covariance matrices and a priori probabilities then become more important. If this is the case then we might have to conclude that the basis of our discrimination is poor however. The second important underlying notion that is important to the search for a significant difference in the means is the dispersion of the variables. If the means are well separated and the respective standard deviations are relatively small then the variables will provide significant discrimination.

The MDA modelling of corporate distress will only be successful to the extent that there is a significant difference between the individual group means of the original data in the model. Without this phenomenon, all else is a complete waste of time. Multivariate normality, nor the equality of the variance-covariance matrices, nor the correct usage of *a priori* adjusted cut-off points will save the particular research from being a futile pursuit of insignificance. All else pales.

VI: Intertemporal Validation:

It is with a good measure of uncertainty that I suggest that intertemporal validation is possibly the sixth critical issue. This is because there is a sense in which any data held back from the sample is a *hold-out* sample, and, I have already dismissed the *hold-out* method as being both pseudo-scientific and an inefficient use of data. I think that I would resolve my dilemma by arguing that in any model building exercise we should use all of the available data for the reasons outlined earlier. It is probable that the process of model construction will take some time. This elapsed time period will probably provide more data for *ex ante* validation. If carefully specified models do not validate intertemporarily then I suggest that it is more appropriate to search for reason why this might be so rather than merely data dredge for another variable.

ISSUES OF A LESSER KIND:

Perhaps not so much *"much ado about nothing"* as a question of the relative importance of a few issues is at stake here. Having analysed the first sixth critical issues suggested in this chapter, the question of the multivariate normality of the distribution of the data, the equality of the variance-covariance matrices and *a priori* probabilities needs to be examined for their relative importance in the modelling process. I suggest that not only are these issues are seventh, eighth and ninth in ranking, but that the measurement scale is ordinal, not interval, for they are of much lesser importance. Much research has been carried out to show that many models appear to classify satisfactorily, despite the breach of the assumptions, yet they have little time-robustness. This failure to stand the test of time derives from the first six critical issues rather than the remaining three issues whose importance this research suggests has been over-emphasized in the literature.

VII: Multivariate Normality.

The seventh critical issue is the question of multivariate normality. As is well known multivariate normality is one of the major pillars upon which Fisher developed his multiple linear discriminant function. The conventional wisdom in the literature can be classified in either of two ways. There are those who argue that unless ratio data for failed and non-failed companies is multivariately normally distributed the resultant MDA models are highly likely to be unsatisfactory because probabilities of group membership are difficult, if not impossible to estimate with any degree of confidence. On the other hand, numerous applied or empirical researchers have ignored the this requirement and found that their models seem to classify their particular data quite adequately. The most common empirical research practice has been to ignore the question of testing for multivariate normality and simply run the ratio data for failed and non-failed companies through a particular MDA computer programme and observe the extent to which the model correctly classifies the companies. The extent to which the multivariate normality assumption is critical, is not so much in the determination of the discriminant coefficients, but in the probabilities of classification and the extent to which we are able to make statements about the significance of the difference between the centroids of the z-scores of the failed and non-failed companies. Obviously, without any knowledge of the distributional properties, we are unable to make satisfactory statements about the probability of a particular company belonging to the failed and non-failed groups.

However desirable it would seem to be to have all ratio data multivariately normally distributed, it appears that such a quality is almost always elusive. We are highly unlikely to find convenient ratio or other financial accounting data with such properties. Watson [1990] does provide some hope in this regard by using transformations but we might have cause to reflect upon the extent to which highly transformed data can be said to reflect observable phenomena. Some researchers might well find grounds for resisting complex transformations of easily measurable phenomena merely to conform to some vague notion of multivariate normality.

If researchers find that their data is not multivariately normally distributed, or that it is undesirable or impossible to transform into new variables with such properties, then all is not necessarily lost. The findings of chapter two, and particularly chapter four, show that even if data is not appropriately distributed it might be quite satisfactorily used to develop MDA models of corporate distress classification. The graph sequences in chapter two show that provided mean differences are sufficiently large, then the question of the distribution of the original data becomes of lesser importance. There is a trade-off between desirable distributional properties and the magnitude of mean differences. It is only where distributions overlap that the relative probabilities are of critical importance. For example, if the mean current ratios for failed and non-failed companies are not markedly different then such a ratio will only add marginal significance to distinguishing between the two groups. On the other hand, if the mean debt ratio for failed companies is 90% and the mean debt ratio for non-failed companies is 10% and there is no group overlap on this particular ratio, then I would argue that the question of multivariate normality is almost, if not, irrelevant. The greater the significance of the distance between the respective mean ratios the more satisfactory our MDA models will be irrespective of the distributional properties of the raw ratio data.

It seems that it is highly unlikely that we will find ratio data that is multivariately normally distributed. This does not necessarily limit our ability to develop a linear discriminant model. This reduces our ability to classify by way of relative probabilities, for without any knowledge of the respective distributions we are unable to compare the respective ordinates by which the probabilities of group membership are established. Compensating for a lack of perfection in the distributional properties of the data is the significance of the difference between the mean ratios. The more marked are the differences between the mean ratios of failed and non-failed companies the less critical this issue becomes.

What can we say to empirical researchers who wish to develop MDA models of corporate distress? Firstly, test for multivariate normality with the appropriate statistical tests. Secondly, report the findings of such tests in any subsequent publication so that interested readers can make appropriate judgements. Thirdly, focus attention upon identifying ratios and other information whose means for failed and non-failed companies are significantly different. The greater the significance of the difference between the means the greater will be the compensation for any breach of the statistical assumptions.

VIII: Equality of the Variance - Covariance Matrices.

The equality of the variance - covariance matrices was another of the pillars upon which Fisher founded his multiple linear discriminant model for distinguishing between two or more groups. As well as investigating the extent to which it is possible to discriminate between failed and non-failed companies when the data is not multivariately normally distributed, the question of the extent to which it was possible to classify correctly when the respective variance - covariance matrices were unequal was also investigated in chapter four. This equality assumption is important for MDA classification procedures for if this condition is not satisfied then the standard deviations of the z-scores of the two groups will not be equal. Again, however, there is a trade-off equality and the significance of the difference between the respective group mean ratios. As the group mean ratios are separated MDA models classify successively better. This is the key to improved classification.

IX: A Priori Probabilities.

Finally, the relative importance of the use of *a priori* probabilities needs to placed in an appropriate context. Chapter two attempted to show the extent of the importance of the use of *a priori* probabilities. Clearly such probabilities are of importance to the scientific establishment of cut-off points, and to the estimation of the population distributions of the z-scores when the variance - covariance matrices are unequal. As the significance of the difference between respective group means on the original data become greater the lesser of an issue *a priori* probabilities becomes. It is really only where the differences are insignificant that fine distinctions become critical. If this is the case then we might well be advised to reassess the extent to which we have an adequate theory of corporate failure. It would seem that it is likely to be more profitable to spend additional research energy pursuing the matter of the significance of the difference between the means than to refine the measurement of *a priori* probabilities.

CONCLUSION:

The results of this investigation into why multiple linear discriminant models of corporate distress themselves rapidly find themselves to be in a distressed condition are surprisingly straight forward. The study began as an investigation into some of the more complex statistical issues yet understanding the conclusions reached does not require a sophisticated multivariate statistical background. The most important issues in the MDA modelling process are those relating to model specification, sample selection, the efficient use of data, the ultimate drive for both real and stable differences between the variables selected, and intertemporal validation. The more complex issues of multivariate normality, the equality of the variance-covariance matrices, and *a priori* probabilities should not be allowed to override the importance of the basic scientific processes that should be involved.

Appendix 4:1

Four Ratio Case

Percentage Correct Classifications by Mean and Variance - Covariance Matrix Differences.

Run No.1.

M.D.	0	0.25	0.50	0.75	1.0	1.25	1.5	1.75	2.0	2.25	2.50	2.75	3.0
1:1	50	53	64	74	81	87	92	95	97	99	99	100	100
2:1	50	49	56	68	76	83	88	91	94	96	98	99	99
3:1	50	49	51	62	71	79	85	90	92	94	96	97	98
4:1	50	49	49	57	67	75	83	87	91	93	95	97	98

Run No.2.

1:1	50	53	65	75	82	88	92	95	97	99	99	100	100
2:1	50	49	58	68	76	84	89	93	95	97	98	98	99
3:1	50	48	53	62	72	80	86	89	93	95	96	97	98
4:1	50	49	49	58	67	76	84	88	91	93	95	96	97

A: 2

Run No.3.													
1:1	50	55	67	76	84	90	93	96	98	99	100	100	100
2:1	50	50	59	68	78	85	90	93	95	98	99	99	100
3:1	50	48	53	63	73	81	87	91	93	96	97	98	99
4:1	50	49	49	59	68	78	84	89	92	94	96	97	98

Run No.4.													
1:1	50	54	68	78	85	90	94	97	98	99	100	100	100
2:1	50	49	60	71	79	87	91	94	96	97	98	99	99
3:1	50	48	54	66	76	83	89	92	95	96	97	98	98
4:1	50	48	50	61	72	81	87	91	94	95	96	97	98

Run No.5.													
1:1	50	55	65	74	83	90	94	97	98	99	99	100	100
2:1	50	50	57	67	76	83	90	94	96	97	98	99	100
3:1	50	48	53	64	72	80	87	91	94	96	97	97	99
4:1	50	48	49	59	68	77	83	89	92	94	96	97	97

A: 3

Run No.6.													
1:1	50	55	65	75	83	89	93	96	98	99	99	100	100
2:1	50	49	57	68	77	84	89	93	96	98	98	99	100
3:1	50	48	52	63	73	81	86	91	94	96	97	98	99
4:1	50	49	49	59	69	77	83	88	92	94	95	96	98

Run No.7.													
1:1	50	54	67	77	83	90	95	97	98	99	100	100	100
2:1	50	49	60	71	79	85	90	94	96	98	99	99	100
3:1	50	48	54	65	76	82	88	92	94	96	97	98	99
4:1	50	48	50	61	72	79	86	90	93	94	96	97	98

Run No.8.													
1:1	50	53	66	74	82	88	92	96	97	99	99	100	100
2:1	50	49	57	67	76	83	89	92	95	97	98	99	99
3:1	50	49	52	63	70	79	85	90	93	95	96	97	98
4:1	50	49	48	57	67	76	82	87	91	94	96	96	97

Run No.9.													
1:1	50	54	66	75	82	88	93	95	97	98	99	99	100
2:1	50	49	60	69	77	84	90	92	95	97	98	99	99
3:1	50	49	52	64	73	80	87	90	94	95	97	98	98
4:1	50	49	48	60	69	77	83	89	91	94	95	96	98

Run No.10.													
1:1	50	54	65	75	82	89	93	96	98	99	99	100	100
2:1	50	49	58	69	77	85	88	93	95	97	98	99	99
3:1	50	48	52	63	73	81	86	90	93	95	97	97	98
4:1	50	48	49	59	70	77	84	88	92	94	95	96	97

M.D. = The standardised mean differences between the ratios of failed and non failed companies.

1:1 = The variance - covariance matrices of failed and non-failed companies are equal.

3:1 = The variance - covariance matrix of the failed companies is three times as large as that of non-failed companies.

Appendix 4:2

Mean Percentages of Correct Classifications

Mean Results

M.D.	0	0.25	0.50	0.75	1.0	1.25	1.50	1.75	2.0	2.25	2.50	2.75	3.0
The Two Ratio Case													
1:1	50	51	59	67	74	80	84	88	91	94	96	97	98
2:1	50	49	52	60	67	74	79	84	88	91	93	95	96
3:1	50	50	48	54	62	69	75	80	85	88	91	93	95
4:1	50	50	47	50	58	65	71	77	82	86	89	91	93
The Three Ratio Case													
1:1	50	53	63	72	79	85	89	93	96	98	99	99	100
2:1	50	49	55	65	73	80	85	90	93	95	97	98	99
3:1	50	49	50	60	68	76	82	87	90	93	95	96	97
4:1	50	49	48	55	64	72	79	84	88	91	93	95	96

Appendix 4:3

Standard Deviations of the Simulated Mean Ratios.

M.D.	0	0.25	0.50	0.75	1.0	1.25	1.50	1.75	2.0	2.25	2.50	2.75	3.0
The Two Ratio Case.													
1:1	0.00	0.40	0.94	1.17	1.37	1.10	0.98	0.80	0.75	0.70	0.54	0.49	0.46
2:1	0.00	0.46	0.49	1.36	1.45	1.50	1.35	1.12	0.92	0.92	0.75	0.54	0.45
3:1	0.00	0.50	0.45	0.70	1.36	1.72	1.47	1.49	1.11	1.18	0.98	0.89	0.50
4:1	0.00	0.30	0.54	0.30	1.43	1.74	1.85	1.74	1.25	1.42	1.40	1.00	0.70
The Three Ratio Case													
1:1	0.00	0.78	0.89	0.92	0.94	0.78	0.66	0.75	0.60	0.78	0.60	0.40	0.40
2:1	0.00	0.63	0.92	1.40	1.33	1.10	1.04	0.81	0.80	0.75	0.64	0.70	0.75
3:1	0.00	0.30	0.70	1.11	1.00	1.04	1.11	0.92	0.70	0.49	0.75	0.45	0.64
4:1	0.00	0.30	0.60	0.98	1.40	1.27	1.11	1.17	1.00	0.64	0.75	0.60	0.45

The Four Ratio Case

0.00	0.42	1.47	1.42	1.20	1.23	0.83	0.99	0.67	0.63	0.42	0.47	0.50
0.00	0.40	1.40	1.36	1.14	1.19	0.92	0.94	0.64	0.60	0.40	0.45	0.49
0.00	0.46	0.92	1.20	1.81	1.20	1.20	0.92	0.81	0.66	0.46	0.50	0.49
0.00	0.49	0.63	1.34	1.81	1.62	1.45	1.20	0.94	0.54	0.50	0.50	0.49

The Ten Ratio Case

0.00	1.00	0.98	0.87	0.90	0.64	0.45	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.98	1.10	1.20	1.30	0.80	0.60	0.30	0.30	0.00	0.00	0.00	0.00
0.00	0.49	1.37	1.36	1.20	0.98	0.40	0.54	0.00	0.49	0.00	0.00	0.00
0.00	0.45	1.27	1.36	1.20	1.20	0.70	0.60	0.30	0.40	0.46	0.40	0.00

= The standardised mean differences between the ratios of failed on-failed companies.

The variance - coviariance matrices of failed and non-failed anies are equal.

The variance - covariance matrix of the failed companies is three as large as that of non-failed companies.

Appendix 4:4

Basic Language Simulation Programme

```
10 PRINT "Linear Discriminant Analysis Simulation"
20 G =2
30 PRINT
40 DIM M(G)
50 PRINT "Number of Failed Companies"
60 INPUT M(1)
70 M(2) = M(1)
80 M = M(1) + M(2)
85 PRINT "Number of 25% Mean differences"
90 INPUT NMS
100 PRINT "Number of Variables"
110 INPUT N
120 DIM P(G), D(N,M), S(N,N), X(N), T(N), C(N+1,G), A(N,G), F(G)
130 P(1) = .5
140 P(2) = 1 - P(2)
150 PRINT "Minimum Variance-Covariance Difference Required"
160 INPUT NC
170 PRINT "Maximin Variance-Covariance Difference Required"
180 INPUT NCOV
185 DIM CMAT(NCOV,NMS)
190 REM
200 PRINT
210 PRINT "System Generating Random Ratios for Non-Failed Co
220 REM
230 PRINT
240 RANDOMIZE
250 K1 = 12
260 FOR J = 1 TO M/2
270 FOR I = 1 TO N
280 A = 0
290 FOR K = 1 TO K1
300 A = A + RND(1)
310 NEXT K
320 D(I,J) = A
330 NEXT I
340 NEXT J
350 PRINT
360 PRINT "System Standardizing Non-Failed Company Ratios"
370 DIM XBAR(N), SD(N)
380 FOR J = 1 TO M/2
390 FOR I = 1 TO N
400 XBAR(I) = XBAR(I) + D(I,J)
410 NEXT I
420 NEXT J
430 FOR I = 1 TO N
440 XBAR(I) = XBAR(I)/(M/2)
450 NEXT I
460 FOR J = 1 TO M/2
```

```
470 FOR I = 1 TO N
480 SD(I) = SD(I) + (D(I,J) - XBAR(I))^2
490 NEXT I
500 NEXT J
510 FOR I = 1 TO N
520 SD(I) = (SD(I)/(M/2 -1))^.5
530 NEXT I
540 FOR J = 1 TO M/2
550 FOR I = 1 TO N
560 D(I,J) = (D(I,J)-XBAR(I))/SD(I)
570 NEXT I
580 NEXT J
590 REM
600 REM Loop through to Increase Variance - Covariance Diff
610 REM
620 PRINT "System Generating FAILED Company Ratios"
630 FOR NRR = NC TO NCOV
640 REM
650 REM Loop through to Increase Mean Ratio Differences
660 SM = -.25
670 FOR NM = 1 TO NMS
680 E = 0
690 SM = SM + .25
700 FOR J = (M/2+1) TO M
710 FOR I = 1 TO N
720 D(I,J) = D(I,J-M/2)*(NRR+1)^.5+SM
730 NEXT I
740 NEXT J
750 PRINT
760 PRINT "NOW computing the Discriminant Model for"
770 PRINT "Variance-Covariance Difference = ";NRR
780 PRINT "Mean Vector Difference         = ";SM
790 REM
800 REM Initialise Vectors and Matrices.
810 T = 0
820 FOR I = 0 TO N
830 X(I) = 0
840 T(I) = 0
850 FOR K = 0 TO N
860 S(I,K) = 0
870 NEXT K
880 NEXT I
890 FOR I = 0 TO G
900 F(G) = 0
910 FOR K = 0 TO N
920 C(K,I) =0
930 A(K,I) = 0
940 NEXT K
```

```
950 C(N+1,I)=0
960 NEXT I
970 REM
980 REM  Compute mean vectors and Covariance Matrices.
990 REM
1000 FOR K = 1 TO G
1010 FOR J = 1 TO M(K)
1020 FOR I = 1 TO N
1030 X(I)= D(I,J+M(1)*(K-1))
1040 A(I,K) = A(I,K) + X(I)
1050 IF K=1 AND J=1 THEN T(I)=X(I)
1060 NEXT I
1070 FOR I = 1 TO N
1080 FOR L = 1 TO I
1090 S(L,I)= S(L,I) + (X(L) - T(L))*(X(I)-T(I))
1100 NEXT L
1110 NEXT I
1120 NEXT J
1130 NEXT K
1140 FOR K = 1 TO G
1150 FOR J = 1 TO N
1160 FOR I = 1 TO J
1170 S(I,J) = S(I,J)-(A(I,K)-M(K)*T(I))*(A(J,K)-M(K)*T(J))/M(K)
1180 NEXT I
1190 NEXT J
1200 T = T+M(K)
1210 NEXT K
1220 FOR J = 1 TO N
1230 FOR I = 1 TO J
1240 S(I,J) = S(I,J)/(T-G)
1250 S(J,I)= S(I,J)
1260 NEXT I
1270 NEXT J
1280 FOR K = 1 TO G
1290 FOR I = 1 TO N
1300 A(I,K) = A(I,K)/M(K)
1310 NEXT I
1320 NEXT K
1330 REM
1340 REM PRINT OUT THE MEAN VECTORS and COVARIANCE MATRICES.
1350 REM
1360 FOR K = 1 TO G
1370 PRINT
1380 IF K>1 THEN GOTO 1410
1390 PRINT " FAILED COMPANIES' MEANS"
1400 GOTO 1420
1410 PRINT " NON-FAILED COMPANIES' MEANS"
1420 FOR I = 1 TO N
1430 IF ABS(A(I,K))<.005 THEN A(I,K) = 0
```

```
1440 PRINT USING "variable ##  ####.##";I;A(I,K)
1450 NEXT I
1460 NEXT K
1470 PRINT
1480 PRINT "Pooled VARIANCE - COVARIANCE MATRIX"
1490 FOR I = 1 TO N
1500 PRINT
1510 FOR J = 1 TO I
1520 PRINT TAB((J-1)*10);INT(S(I,J)*100)/100;
1530 NEXT J
1540 PRINT
1550 NEXT I
1560 PRINT
1570 REM
1580 REM CALCULATE THE DISCRIMINANT FUNCTION.
1590 REM
1600 FOR K = 1 TO N
1610 P = S(K,K)
1620 FOR J = K TO N
1630 IF ABS(P)>ABS(S(J,K)) THEN GOTO 1660
1640 I = J
1650 P = S(J,K)
1660 NEXT J
1670 IF K=I THEN GOTO 1730
1680 FOR J= 1 TO N
1690 P = S(K,J)
1700 S(K,J)=S(I,J)
1710 S(I,J) = P
1720 NEXT J
1730 FOR I = 1 TO N
1740 IF I = K THEN GOTO 1790
1750 FOR J = 1 TO N
1760 IF J = K THEN GOTO 1780
1770 S(I,J) = S(I,J)-S(I,K)*S(K,J)/S(K,K)
1780 NEXT J
1790 NEXT I
1800 FOR I = 1 TO N
1810 IF I = K THEN GOTO 1840
1820 S(K,I)= S(K,I)/S(K,K)
1830 S(I,K) = S(I,K)/S(K,K)
1840 NEXT I
1850 S(K,K)= -1/S(K,K)
1860 NEXT K
1870 FOR K = 1 TO G
1880 FOR I = 1 TO N
1890 C(I,K) = 0
1900 FOR J = 1 TO N
1910 C(I,K) = C(I,K)- A(J,K)*S(I,J)
1920 NEXT J
```

```
1930 NEXT I
1940 FOR I = 1 TO N
1950 C(N+1,K) = C(N+1,K) + C(I,K)*A(I,K)
1960 NEXT I
1970 C(N+1,K) = -C(N+1,K)/2
1980 NEXT K
1990 REM
2000 REM Classify Results
2010 REM
2020 FOR K = 1 TO G
2030 FOR J = 1 TO M(K)
2040 FOR I = 1 TO N
2050 X(I) = D(I,J+M(1)*(K-1))
2060 NEXT I
2070 FOR H = 1 TO G
2080 F(H) =0
2090 FOR I = 1 TO N
2100 F(H) = F(H) +X(I)*C(I,H)
2110 NEXT I
2120 F(H) = F(H) + C(N+1,H)+LOG(P(H))
2130 NEXT H
2140 L = 1
2150 P = F(1)
2160 FOR H = 2 TO G
2170 IF F(H) > P THEN P=F(H):L = H
2180 NEXT H
2190 IF L<>K THEN E=E+1
2200 NEXT J
2210 NEXT K
2220 PRINT
2230 PRINT USING "####.## % Misclassified"; E*100/M
2240 PRINT "*****************************************************
2245 CMAT(NRR,NM)= 100-(E*100/M)
2250 NEXT NM
2260 NEXT NRR
2280 FOR J = 0 TO NMS
2285 INPUT Y
2290 PRINT J,CMAT(0,J),CMAT(1,J),CMAT(2,J),CMAT(3,J)
2300 NEXT J
3000 STOP
```

Appendix 5:1

Increased Errors from Hold-Out Samples from Finite Populations

[[Standard error of sub-sample - standard error of sample]/[Standard error of the sample] - 1.] *100

$$\left\{ \left[\frac{S/[n/a]^{1/2} * [(N - n/a)/(N - 1)]^{1/2}}{S/n^{1/2} * [(N - n)/(N - 1)]^{1/2}} \right] -1 \right\} * 100$$

Simplifies to:

$$\{\{[\{[N-n/a]/[N-1]\}/(n/a)]^{1/2}/[\{[N-n]/[N-1]\}/(n)]^{1/2}\}-1\} * 100$$

which simplifies to:

$$\{[[a(N-n/a)]/(N-n)]^{1/2} -1\} * 100$$

which simplifies to:

$$\{[a(N-n/a)/(N-n)]^{1/2} -1\}* 100$$

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